Classification of Linear Codes with Prescribed Minimum Distance and New Upper Bounds

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Introduction

Motivation

- Gaps between lower and upper bounds – http://codetables.de. (Show existence or nonexistence for the upper bound)
- Full classification of linear codes having certain parameters. (There is no self-dual $[72, 36, 16]_2$-code with automorphism of order 7! – joint work with G. Nebe)

Inspired by

- Y. Edel, J. Bierbrauer, Inverting construction Y_1, IEEE Transactions on Information Theory, 44, 1993, (1998)
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n/k	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
1	1																			
2	2	1																		
3	3	2	1																	
4	4	3	2	1																
5	5	4	3	2	1															
6	6	4	4	2	2	1														
7	7	5	4	3	2	2	1													
8	8	6	5	4	3	2	2	1												
9	9	7	6	5	4	3	2	2	1											
10	10	8	6	6	5	4	3	2	2	1										
n/k	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
11	11	8	7	6	6	5	4	3	2	2	1									
12	12	9	8	7	6	6	4	4	3	2	2	1								
13	13	10	9	8	7	6	5	4	4	3	2	2	1							
14	14	11	10	9	8	7	6	5	4	4	3	2	2	1						
15	15	12	11	10	8	8	7	6	5	4	4	3	2	2	1					
16	16	12	12	11	9	8	8	7	6	5	4	4	3	2	2	1				
17	17	13	12	12	10	9	8	8	7	6	5	4	4	3	2	2	1			
18	18	14	13	12	10	10	9	[illegible]	8	6	6	5	4	3	3	2	2	1		
19	19	15	14	12	11	10	9	8-9	8	7	6	6	5	4	3	3	2	2	1	
20	20	16	15	13	12	11	10	[illegible]	8-9	8	7	6	6	5	4	3	3	2	2	1
n/k	1	2	3	4	5	6	7	8	9	10	11	12	13	[illegible]	15	16	17	18	19	20
21	21	16	16	14	13	12	11	10	9	8-9	7-8	7	6	5-6	5	4	3	3	2	2
22	22	17	16	15	14	12-13	12	11	10	9	8-9	7-8	6-7	[illegible]	5-6	4-5	4	3	2	2
23	23	18	16	16	15	13	12-13	12	11	10	9	8-9	7-8	6-7	6	5-6	4-5	4	3	2
24	24	19	17	16	16	14	13	12-13	12	11	10	9	8-9	7-8	6-7	6	5-6	4-5	4	3
25	25	20	18	17	16	15	14	12-13	12-13	12	11	10	9	8-9	7-8	6-7	6	5-6	4-5	4
26	26	20	19	18	16	16	14-15	13-14	12-13	12-13	12	11	10	9	8-9	7-8	6-7	6	5-6	4-5
27	27	21	20	19	17	16	15-16	14-15	13-14	12-13	12-13	12	11	10	9	7-9	6-8	6-7	6	5-6
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]

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Basics

Notation

Let C be an $[n, k, d]_q$-code. If $C^\perp$ has minimum distance $d^\perp$ we also write $[n, k, d]_q^{d^\perp}$.

Semilinear Mappings

A mapping $\sigma : \mathbb{F}_q^n \to \mathbb{F}_q^n$ is called semilinear, if there exists some $\alpha \in \mathrm{Aut}(\mathbb{F}_q)$ with

- $\sigma(u + v) = \sigma(u) + \sigma(v)$
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Code Equivalence

Two $[n, k, d]_q^{d^\perp}$-codes C, C' are equivalent $\iff$ exists some semilinear isometry ι with $\iota(C) = C'$.

Equivalence of matrices

Similarly we say that generator (parity check) matrices are equivalent if they represent equivalent codes.

Transversal of equivalence classes

$T(n, k, d, d^\perp, q)$ denotes a complete set of non-equivalent parity check matrices of all $[n, k, \geq d]_q^{d^\perp}$-codes.

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A canonical form algorithm

Unique Orbit Representatives

With the help of the algorithm

T. Feulner, The Automorphism Groups of Linear Codes and Canonical Representatives of Their Semilinear Isometry Classes, Advances in Mathematics of Communication, 3, 363-383, (2009)

we can compute unique orbit representatives and hence determine $T(n, k, d, d^{\perp}, q)$ from a superset $\mathcal{T}(n, k, d, d^{\perp}, q)$ very efficiently.

Problem

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Construction Y_1

Let C be an $[n, k, d]_q^{d^\perp}$-code. Then there exists an $[n - d^\perp, k - d^\perp + 1, \geq d]_q^{\geq \left\lceil \frac{d^\perp}{q} \right\rceil}$-code.

Proof.

Without loss of generality C has a parity check matrix of the following form

$$\Delta := \left(\begin{array}{c|c} \Delta' & X \\ \hline 0_{n-d^\perp} & c \end{array} \right)$$

with $(0_{n-d^\perp}, c) \in C^\perp$ a codeword of minimum distance $\mathrm{wt}(c) = d^\perp$. The code with parity check matrix Δ' has got the desired parameters. □

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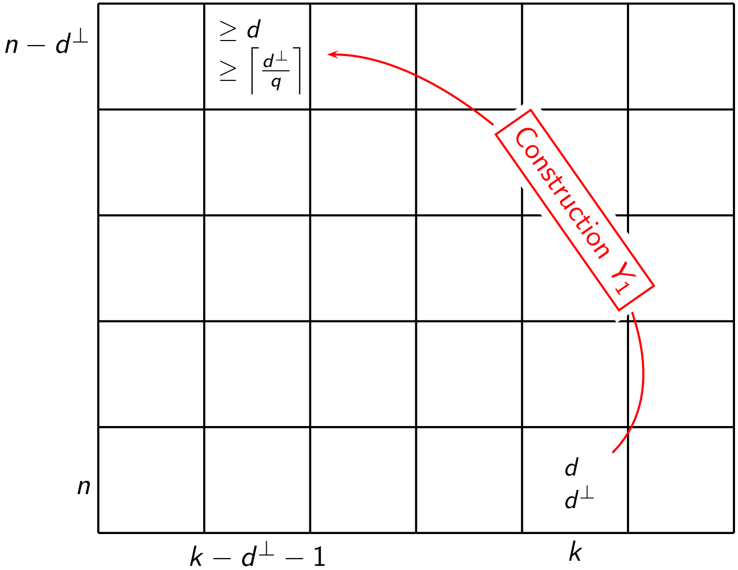
Inverting Construction Y_1



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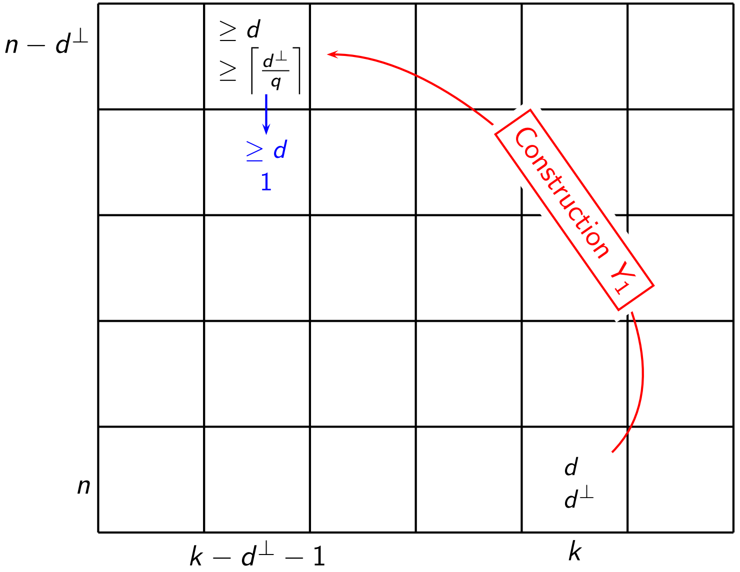

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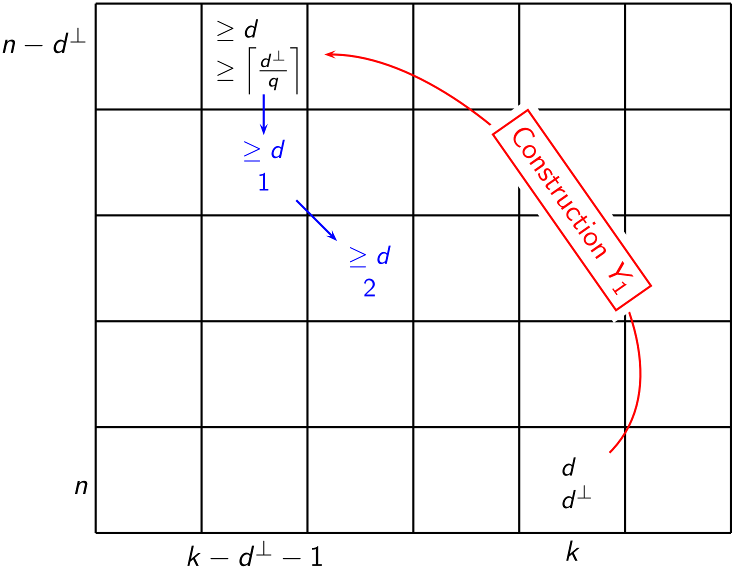

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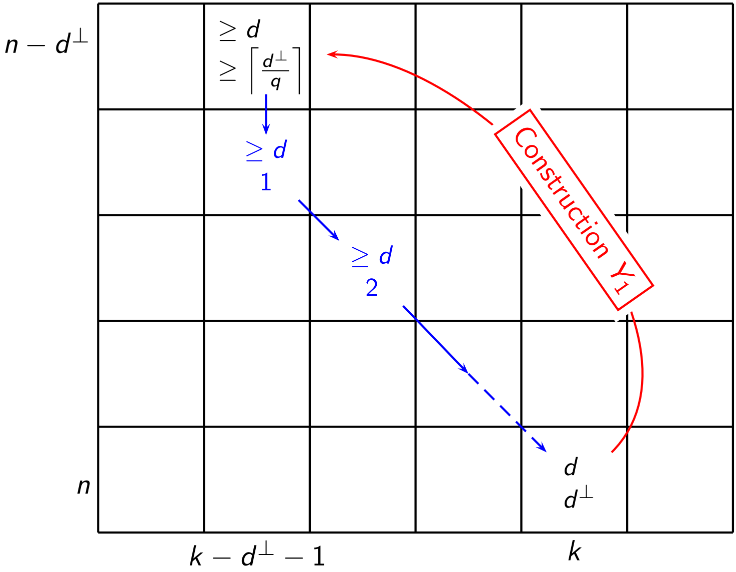

Inverting Construction Y_1

Iteration Starting Point

Let S be an arbitrary transversal of parity check matrices of all $[n-d^\perp, k-d^\perp+1, \geq d]_q^{\geq \left\lceil \frac{d^\perp}{q} \right\rceil}$-codes.

Existence of predecessors

Each equivalence class of parity check matrices of the $[n, k, \geq d]_q^{d^\perp}$-codes contains at least one matrix

$$\widetilde{\Delta} = \begin{pmatrix} \Delta' & X \\ 0_{n-d^\perp} & 1_{d^\perp} \end{pmatrix}$$

with

- $\Delta' \in S$
- $X \in \mathbb{F}_q^{(n-k-1)\times d^\perp}$ with lexicographically ordered columns

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A special transversal

Choosing the smallest matrix

$$\widetilde{\Delta} = \begin{pmatrix} \Delta' & X \\ 0_{n-d^\perp} & 1_{d^\perp} \end{pmatrix}$$

in each equivalence class defines a transversal $T(n, k, d, d^\perp, S, q)$.

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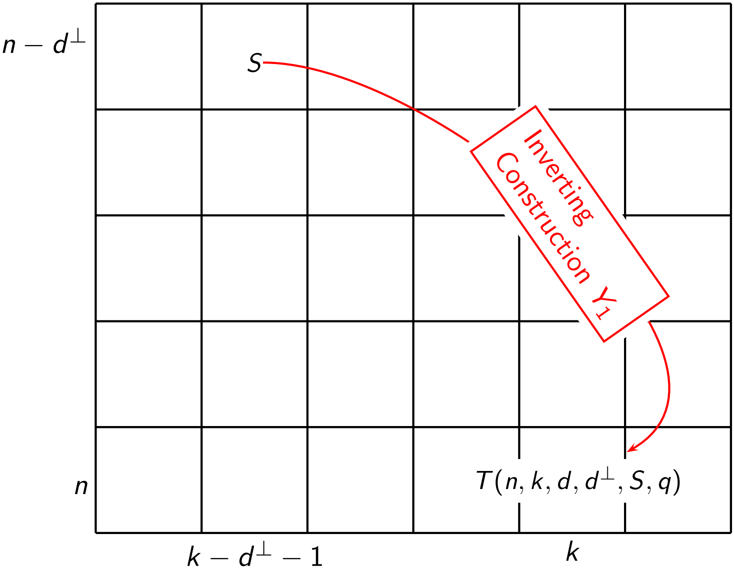

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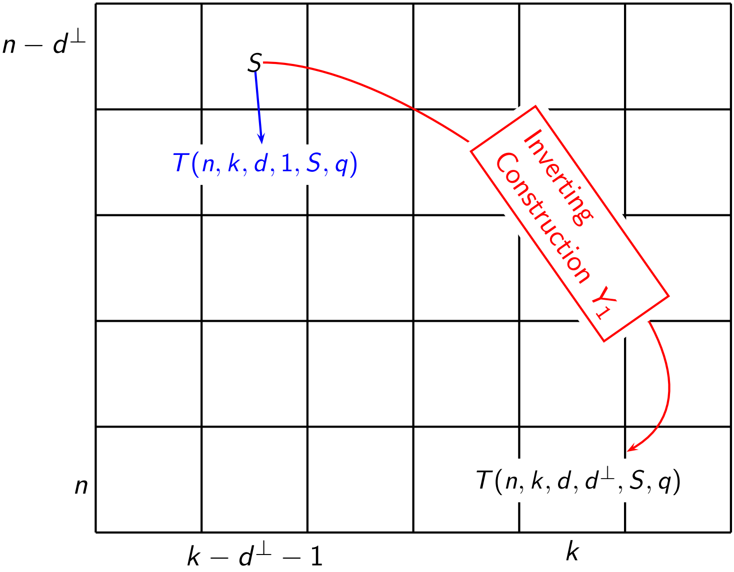

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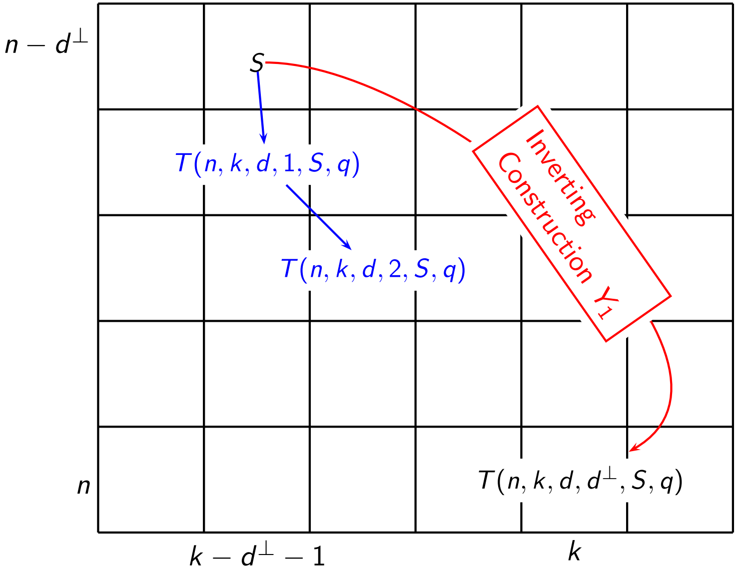

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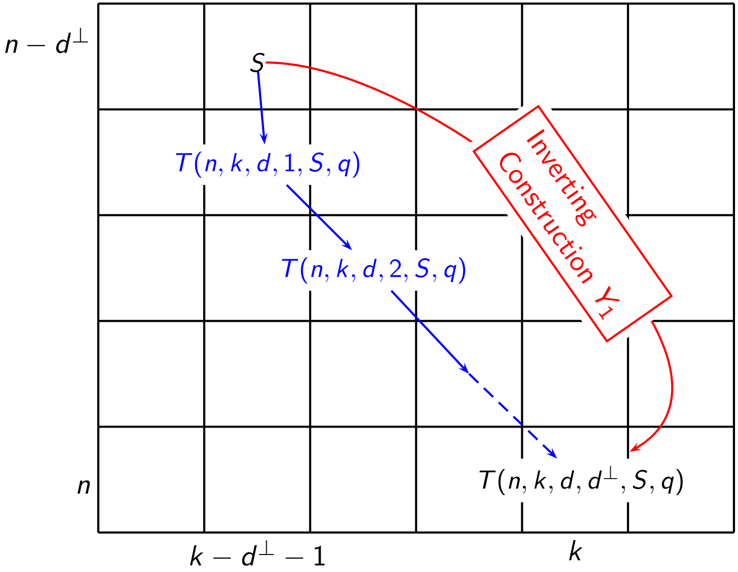

Details

Computation of $T(n,k,d,1,S,q)$

- Define $\mathcal{T}(n,k,d,1,S,q) := \left\{ \begin{pmatrix} \Delta' & 0 \\ 0_{n-d^\perp} & 1 \end{pmatrix} | \Delta' \in S \right\}$
- Filter $\mathcal{T}(n,k,d,1,S,q)$ for nonisomorphic copies

Computation of $T(n,k,d,d^\perp,S,q)$, $d^\perp \geq 2$

- Compute $\mathcal{T}(n,k,d,d^\perp,S,q)$: For all

$$\begin{pmatrix} \Delta' & 0 & x_1 & \cdots & x_{d^\perp-2} \\ 0_{n-d^\perp} & 1 & 1 & \cdots & 1 \end{pmatrix} \in T(n-1,k-1,d,d^\perp-1,S,q)$$

add all possible columns $\binom{x_{d^\perp-1}}{1}$ with $x_{d^\perp-1} \geq x_{d^\perp-2}$ which fulfills the conditions on d and $d^\perp$.
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An Example: Does a $[21, 14, 6]_4$-code exist?

From http://codetables.de we determine that $d^\perp \in \{9, 10, 11\}$.
The following table gives the number of equivalence classes for $d \geq 6$, distinguished by $d^\perp$:

n	$n-k=6$	$n-k=7$
10		
11		
12	$1^0 \ldots 5^0\ 6^1$	
13		1^1
14		2^2
15		3^7
16		4^{13}
17		5^9
18		6^5
19		7^1
20		8^1
21		9^0

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13		1^1	2^6
14		2^2	3^{30}
15		3^7	4^{88}
16		4^{13}	5^{64}
17		5^9	6^{17}
18		6^5	7^1
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12	$1^0 \ldots 5^0\ 6^1$		1^1	2^{51}
13		1^1	2^6	3^{1219}
14		2^2	3^{30}	4^{7431}
15		3^7	4^{88}	5^{3797}
16		4^{13}	5^{64}	6^{261}
17		5^9	6^{17}	7^4
18		6^5	7^1	8^0
19		7^1	8^0	9^0
20		8^1	9^0	10^0
21		9^0	10^0	11^0

Results

Nonexistence

There are no codes with parameters

- $[35, 10, 13]_2$
- $[22, 8, 10]_3$, $[24, 14, 7]_3$, $[28, 21, 5]_3$
- $[19, 8, 9]_4$, $[21, 14, 6]_4$, $[22, 16, 5]_4$, $[27, 17, 8]_4$, $[30, 21, 7]_4$, $[39, 27, 9]_4$
- $[16, 5, 10]_5$, $[16, 6, 9]_5$, $[17, 8, 8]_5$
- $[15, 8, 7]_7$, $[26, 20, 6]_7$
- $[30, 23, 7]_8$, $[37, 31, 5]_8$

and 391 derived new upper bounds.

Existence

There is a $[17, 11, 6]_9$-code.

Results

Nonexistence

There are no codes with parameters

- $[35, 10, 13]_2$
- $[22, 8, 10]_3$, $[24, 14, 7]_3$, $[28, 21, 5]_3$
- $[19, 8, 9]_4$, $[21, 14, 6]_4$, $[22, 16, 5]_4$, $[27, 17, 8]_4$, $[30, 21, 7]_4$, $[39, 27, 9]_4$
- $[16, 5, 10]_5$, $[16, 6, 9]_5$, $[17, 8, 8]_5$
- $[15, 8, 7]_7$, $[26, 20, 6]_7$
- $[30, 23, 7]_8$, $[37, 31, 5]_8$

and 391 derived new upper bounds.

Existence

There is a $[17, 11, 6]_9$-code.

Thank you for your attention.