Abstract

Erdős-Ko-Rado theorems for dual polar spaces

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The original Erdős-Ko-Rado-theorem was proved in [1]:

Theorem 1 *If S is a family of subsets of size k in a set Ω with $|\Omega| = n$ with $n \geq 2k$, such that no two elements of S are disjoint, then $|S| \leq \binom{n-1}{k-1}$. If $n \geq 2k+1$, this bound can only be reached if S is the family of all subsets of size k, containing a fixed element of Ω.*

Several analogue results of the previous Theorem have been proved so far. In particular, we mention the following q-analogue of Theorem 1:

Theorem 2 *[2, 3, 4] If S is a set of subspaces with algebraic dimension k, in a vector space V with algebraic dimension n over $GF(q)$, with $n \geq 2k$, such that no two elements of S intersect trivially, then $|S| \leq \begin{bmatrix} n-1 \\ k-1 \end{bmatrix}_q$. If $n \geq 2k+1$, then this bound can only be reached if S is the set of all subspaces with algebraic dimension k, containing a fixed subspace with algebraic dimension 1 in V.*

For a proof of the characterization of the sets reaching the bound of Theorem 2, we refer to [3]: the authors give a very short proof considering the associated association scheme. We prove similar results for maximum independent sets in dual polar graphs. In other words, we characterize the sets of maximum size of generators in polar spaces, such that no two intersect trivially.

This is a joint work with Frederic Vanhove and Leo Storme.

References

[1] P. Erdős, C. Ko, and R. Rado. Intersection theorems for systems of finite sets. *Quart. J. Math. Oxford Ser. (2)* **12** (1961), 313–320.

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[3] C. Godsil and M. W. Newman. Independent sets in association schemes. *Combinatorica* , **4** (2006), 431–443.

[4] W. N. Hsieh. Intersection theorems for systems of finite vector spaces. *Discrete Mathematics* , **12** (1975), 1–16.