Abstract

Trellis Representations for Linear Block Codes

Heide Gluesing-Luerssen

University of Kentucky
Department of Mathematics
715 Patterson Office Tower
Lexington, KY 40506
USA

During the last decade conventional and tail-biting trellis representations of linear block codes have gained a great deal of attention. A tail-biting trellis for a block code of length n is an edge-labeled layered graph on the circular time axis $\mathbb{Z}_n$ such that the codewords are the label sequences corresponding to all cycles through the graph (that is, start and endpoint coincide). A conventional trellis is a tail-biting trellis for which the vertex set at time zero is a singleton and thus all paths are cycles. Both types of trellises may give rise to efficient decoding algorithms of Viterbi type. As a consequence, the quest is on for constructing trellises with low complexity with respect to certain measures. For conventional trellises it is by now well-known that for a given linear block code there exists a unique (up to isomorphism) minimal trellis and minimality coincides with biproperness as well as with non-mergeability; see [5, 4, 2]. This unique minimal trellis simultaneously minimizes the vertex count at each time as well as any other conceivable complexity measure. For tail-biting trellises none of the above is true [1, 3]. In [3] a construction is given from which all minimal tail-biting trellises can be derived, but this construction also comprises certain non-minimal trellises. In [6], a construction of tail-biting trellises is presented which generalizes the well-known BCJR-construction of conventional trellises. We will show how these two approaches are related. In particular, we will see that all trellises obtained as in [3] are non-mergeable and isomorphic to those in [6].

References

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