Abstract

Regular Antichains

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This talk is about k-regular antichains on $[m]$. This incidence structure occurs in Sperner theory, in design theory and in extremal set theory. We want to mention some constructions, results and several open problems. We are especially interested in the case $k \leq m$.
Let $\mathcal{B}$ be a subset of $2^{[m]}$ with $[m] := \{1, 2, \dots, m\}$. The size of $\mathcal{B}$ is $n := |\mathcal{B}|$. The collection $\mathcal{B}$ of sets is called an antichain if for all distinct $A, B \in \mathcal{B}$: $A \nsubseteq B$. An antichain $\mathcal{B}$ is called k-regular, if for each $i \in [m]$ there are exactly k blocks $B_1, B_2, \dots, B_k \in \mathcal{B}$ containing i. That is, $|\{B \in \mathcal{B} : i \in B\}| = k$ for all $i \in [m]$. If $\mathcal{B} \subseteq 2^{[m]}$ is a k-regular antichain of size n we call $\mathcal{B}$ a (k, m, n)-antichain.
A Completely Separating System (CSS) is the dual of an antichain. A CSS $\mathcal{C}$ is a collection of subsets of $[n]$ such that for all distinct $a, b \in [n]$ there are sets $A, B \in \mathcal{C}$ with $a \in A - B$ and $b \in B - A$.
The most important problem this talk is about is to decide if a (k, m, n)-antichain exists or not, for given parameters $k, m, n \in \mathbb{N}$.