Abstract

On permutation codes in given permutation groups

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A permutation code (or array) of length n and distance d is a set T of permutations from some fixed set of n symbols such that the Hamming distance between each distinct $x, y \in T$ is at least d. By elementary counting, one has $|T| \leq n(n-1)\cdots(n-d+1)$ and equality holds if and only if T is a sharply $(n-d+1)$-transitive set of permutations. It is well known that sharply 1- and 2-transitive sets of permutations correspond to Latin squares and affine planes, respectively.

We investigate the question of existence of sharply transitive sets in given permutation groups. Using combinatorial and finite geometric methods, we show that

- the Mathieu group M_{22} does not contain a sharply 1-transitive set,
- for q even, the symplectic group $Sp(2n, q)$ does not contain a sharply 1-transitive set,
- for $n \equiv 2, 3 \pmod 4$, the alternating group A_n does not contain a sharply 2-transitive set of permutations.

References

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[2] J. Quistorff. A survey on packing and covering problems in the Hamming permutation space. Electron. J. Combin. 13 (2006), no. 1, Article 1, 13 pp. (electronic).