Constructions of Two-Weight Codes over Rings

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Introduction

There is a known link between **two-weight codes** and **strongly regular graphs**.

1. Delsarte (1972) showed that a code over $\mathbb{F}_q$ with two non-zero Hamming weights yields a strongly regular graph.
2. This result was extended by Byrne, Greferath and Honold (2008) to codes with two non-zero homogeneous weights over finite Frobenius rings satisfying certain conditions.

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  1. Find new constructions of two-weight codes over finite Frobenius rings.
  2. Classify any strongly regular graphs resulting from these codes.

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  1. Find new constructions of two-weight codes over finite Frobenius rings.
  2. Classify any strongly regular graphs resulting from these codes.
- We will present two new constructions.
  1. One resulting from unions of submodules.
  2. One resulting from two-weight rings.

Outline

- Codes over rings
- Strongly regular graphs
- Two-weight codes and strongly regular graphs
- Two new constructions for two-weight codes

Frobenius Rings

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- **Examples of Frobenius Rings**
  1. Finite fields are Frobenius.
  2. $\mathbb{Z}_m$ is Frobenius.
  3. Chain rings are Frobenius.
  4. If R is Frobenius, so is $M_n(R)$.

Definition

A map w: $R \to \mathbb{R}$ is called a (left) **homogeneous weight** if the following hold:

1. $w(0) = 0$.
2. If $Rx = Ry$, then $w(x) = w(y)$ for all x, y in R.
3. There exists a real number $\gamma \geq 0$ such that

$$\sum_{y \in Rx} w(y) = \gamma |Rx|, \text{ for all } x \in R \backslash \{\mathbf{0}\}.$$

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Examples:

1. Over $\mathbb{F}_q$, the **Hamming weight** is homogeneous with $\gamma = \frac{q-1}{q}$.
2. Over $\mathbb{Z}_4$, the **Lee weight** is homogeneous with $\gamma = 1$.

$$\mathbf{w(0) = 0; w(1) = 1; w(2) = 2; w(3) = 1.}$$

The Homogeneous Weight

If R is Frobenius:

1. It has a generating character χ.
2. For $\gamma \in \mathbb{R}$, the (left) **homogeneous weight** is given by

$$w(x) = \gamma \left[1 - \frac{1}{|R^\times|} \sum_{u \in R^\times} \chi(ux) \right].$$

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Example:

- Over $\mathbb{Z}_4$, a generating character χ is given by i.
- Taking $\gamma = 1$,

$$w(x) = 1 - \frac{1}{2}(i^x + i^{3x}).$$

This results in the Lee weight.

Codes Over Rings

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Codes Over Rings

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- A (left) **linear code** C is a submodule of ${}_R R^n$. We write $C \leq {}_R R^n$.
- For $c = (c_1, \ldots, c_n) \in C$, the homogeneous weight of c is given by $w(c) = \sum_{i=1}^{n} w(c_i)$.

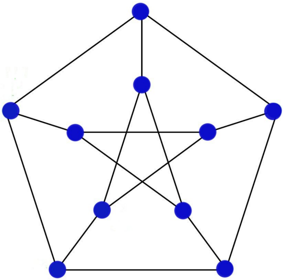
Strongly Regular Graphs

Definition

A graph $G = (V, E)$ with vertex set V and edge set E is **strongly regular** with parameters (N, K, λ, μ) if:

1. G has N vertices.
2. Each vertex is connected to K edges.
3. Every adjacent pair of vertices have exactly λ common neighbours in V.
4. Every non-adjacent pair of vertices have exactly μ common neighbours in V.

Example:
The **Petersen Graph** is strongly regular with parameters $(10, 3, 0, 1)$.



The n^2-graphs

- The $\mathbf{n^2}$ or **lattice graphs** are formed by taking the elements of the set $\{1, 2, \ldots, n\} \times \{1, 2, \ldots, n\}$ as vertices. Two vertices, (x, y) and (x', y') are adjacent if and only if $x = x'$ or $y = y'$.

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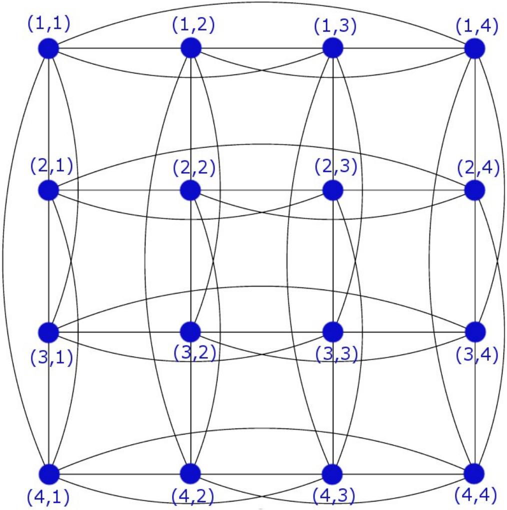
- These graphs are strongly regular with parameters

$$(n^2, 2n - 2, n - 2, 2).$$

The n^2-graphs

The $\mathbf{4^2}$-graph is strongly regular with parameters $(16, 6, 2, 2)$.



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An **orthogonal array** $OA(s,k)$ is an $s^2 \times k$ array with entries from an s-set S, such that in any two columns of the array, each ordered pair of symbols from $S \times S$ occurs exactly once.

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A strongly regular graph can be constructed from an $OA(s, k)$:

- We take the s^2 rows as vertices.
- Two rows or vertices are adjacent if they have a common entry in a column.
- The resulting strongly regular graph has parameters

$$(s^2, sk - s, k^2 - 3k + s, k^2 - k).$$

Graphs from Orthogonal Arrays

Example: This $OA(4,3)$ gives a strongly regular graph $(16,9,4,6)$.

$$\begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 2 & 2 \\ 0 & 3 & 3 \\ 1 & 0 & 1 \\ 1 & 1 & 2 \\ 1 & 2 & 3 \\ 1 & 3 & 0 \\ 2 & 0 & 2 \\ 2 & 1 & 3 \\ 2 & 2 & 0 \\ 2 & 3 & 1 \\ 3 & 0 & 3 \\ 3 & 1 & 0 \\ 3 & 2 & 1 \\ 3 & 3 & 2 \end{pmatrix}$$

Classifying Strongly Regular Graphs

- Two strongly regular graphs with the same parameters are not necessarily isomorphic. The number of non-isomorphic strongly regular graphs for a given parameter set can vary greatly.

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- For $n \neq 4$, the n^2 graphs are unique up to isomorphism (Shrikhande, 1959). So there is exactly 1 strongly regular graph with parameters $(36, 10, 4, 2)$.
- How many non-isomorphic graphs with parameters $(36, 15, 6, 6)$ are there? $32, 548$ (McKay, Spence, 2001).
- In most cases, the number is not known.

Definition

A **two-weight code** $C \leq {}_R R^n$ is a code with the property that its codewords have exactly two non-zero homogeneous weights, w_1 and w_2, with $w_1 < w_2$.

Two-Weight Codes and Strongly Regular Graphs

Definition

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Definition

For a two-weight code C, we define a **graph** $\mathbf{G}(\mathbf{C})$ whose vertices are the codewords of C. Two vertices c and c' are joined if $w(c - c') = w_1$.

Theorem (Byrne, Greferath, Honold)

Let $C \leq {}_R R^n$ be a projective, regular two-weight code. Then $G(C)$ is a strongly regular graph with parameters

$$\begin{aligned} N &= |C|, \\ K &= \frac{(n-w_2)|C| + w_2}{w_1 - w_2}, \\ \lambda &= \frac{nK[1-(1-\frac{w_1}{n})^2] + w_2(1-K)}{w_1 - w_2}, \\ \mu &= \frac{nK[1-(1-\frac{w_1}{n})(1-\frac{w_2}{n})] - w_2 K}{w_1 - w_2}. \end{aligned}$$

Submodules Construction

- There is a well known construction for a family of two-weight codes over $\mathbb{F}_q$ arising from unions of subspaces of $\mathbb{F}_q^k$ (see for example a survey paper by Calderbank and Kantor (1986)). We will generalize this construction.

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- For $M \leq R_R^k$, let $M^{\perp} = \{x \in R^k : x \cdot m = 0 \ \forall \ m \in M\}$.
- Some notation: Let $M_1, M_2, \ldots, M_r \leq R_R^k$. We let

$$(M_1 \backslash \mathbf{0} | M_2 \backslash \mathbf{0} | \ldots | M_r \backslash \mathbf{0})$$

denote the matrix whose columns consist of the non-zero elements of M_1 in some order, followed by the non-zero elements of M_2 and so on.

Submodules Construction

Theorem

Let $M_1, \ldots, M_r, r \geq 2$ be submodules of R_R^k such that

1. $|M_i| = v \; \forall \; i.$
2. $M_i \cap M_j = \mathbf{0} \; \forall \; i, j.$
3. *For every* $x \in R^k, |i : x \in M_i^{\perp}| \in \{0, 1, r\}.$

Let $C = \{x \cdot (M_1 \backslash \mathbf{0} | M_2 \backslash \mathbf{0} | \ldots | M_r \backslash \mathbf{0}) : x \in R^k\}.$

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Let $C = \{x \cdot (M_1 \backslash \mathbf{0} | M_2 \backslash \mathbf{0} | \ldots | M_r \backslash \mathbf{0}) : x \in R^k\}$.
Then C is a two-weight code of order v^2 and length $rv - r$ with

$$w_1 = (r-1)v \text{ and } w_2 = rv.$$

Theorem

Let C be the two-weight code described on the previous slide. Then $G(C)$ is a strongly regular graph with parameters

$$(v^2, rv - r, r^2 + v - 3r, r^2 - r)$$

and is isomorphic to the graph from an orthogonal array $OA(v, r)$ derived from C.

Submodules Construction

Example:

- Let $R^k = \mathbb{Z}_4{}^2$ and let $M_1 =< (0,1) >$, $M_2 =< (1,0) >$, $M_3 =< (1,1) >$.

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- These submodules satisfy our three conditions:
  1. $|M_i| = 4, i = 1,2,3.$
  2. $M_i \cap M_j = \mathbf{0}$.
  3. for every $x \in \mathbb{Z}_4^2, |i : x \in M_i^{\perp}| \in \{0,1,3\}$.

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- Let $C = \{x \cdot \left(\begin{array}{ccc|ccc|ccc} 0 & 0 & 0 & 1 & 2 & 3 & 1 & 2 & 3 \\ 1 & 2 & 3 & 0 & 0 & 0 & 1 & 2 & 3 \end{array}\right) : x \in \mathbb{Z}_4^2\}$.

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- C is a a two-weight code with $w_1 = 8$ and $w_2 = 12$.

Submodules Construction

$$(0,0,0|0,0,0|0,0,0)$$
$$(\mathbf{0},\mathbf{0},\mathbf{0}|\mathbf{1},\mathbf{2},\mathbf{3}|\mathbf{1},\mathbf{2},\mathbf{3})$$
$$(0,0,0|2,0,2|2,0,2)$$
$$(0,0,0|3,2,1|3,2,1)$$
$$(1,2,3|0,0,0|1,2,3)$$
$$(\mathbf{1},\mathbf{2},\mathbf{3}|\mathbf{1},\mathbf{2},\mathbf{3}|\mathbf{2},\mathbf{0},\mathbf{2})$$
$$(1,2,3|2,0,2|3,2,1)$$
$$(1,2,3|3,2,1|0,0,0)$$
$$(2,0,2|0,0,0|2,0,2)$$
$$(2,0,2|1,2,3|3,2,1)$$
$$(2,0,2|2,0,2|0,0,0)$$
$$(2,0,2|3,2,1|1,2,3)$$
$$(3,2,1|0,0,0|3,2,1)$$
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$$
\begin{array}{c}
(0,0,0|0,0,0|0,0,0) \\
(\mathbf{0},\mathbf{0},\mathbf{0}|\mathbf{1},\mathbf{2},\mathbf{3}|\mathbf{1},\mathbf{2},\mathbf{3}) \\
(0,0,0|2,0,2|2,0,2) \\
(0,0,0|3,2,1|3,2,1) \\
(1,2,3|0,0,0|1,2,3) \\
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(3,2,1|1,2,3|0,0,0) \\
(3,2,1|2,0,2|1,2,3) \\
(3,2,1|3,2,1|2,0,2)
\end{array}
\longrightarrow
\begin{array}{c}
(0,0,0) \mapsto 0 \\
(1,2,3) \mapsto 1 \\
(2,0,2) \mapsto 2 \\
(3,2,1) \mapsto 3
\end{array}
$$

Submodules Construction

$$
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(0,0,0|0,0,0|0,0,0)\\
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\longrightarrow
\begin{pmatrix}
0 & 0 & 0\\
\mathbf{0} & \mathbf{1} & \mathbf{1}\\
0 & 2 & 2\\
0 & 3 & 3\\
1 & 0 & 1\\
\mathbf{1} & \mathbf{1} & \mathbf{2}\\
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\end{pmatrix}
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1. The code $C = \{x \cdot (M_1 \backslash \mathbf{0} | M_2 \backslash \mathbf{0} | M_3 \backslash \mathbf{0}) : x \in R^2\}$ is a two-weight code with $w_1 = 2|R|$ and $w_2 = 3|R|$. $G(C)$ is isomorphic to the strongly regular graph from an orthogonal array $OA(|R|, 3)$ that can be constructed from C. It has parameters
$$(|R|^2, 3|R| - 3, |R|, 6).$$

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2. The code $C = \{x \cdot (M_1 \backslash \mathbf{0} | M_2 \backslash \mathbf{0}) : x \in R^2\}$ is a two-weight code $w_1 = |R|$ and $w_2 = 2|R|$. $G(C)$ is a strongly regular graph, isomorphic to the $|R|^2$-graph and has parameters
$$(|R|^2, 2|R| - 2, |R| - 2, 2).$$

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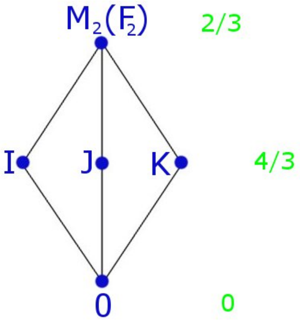
Examples of two-weight rings:

- Chain rings.
- $\mathbb{F}_q \oplus \mathbb{F}_q$.

$\mathbf{M_2}(\mathbb{F}_\mathbf{2})$ **:** Consider the left ideal lattice of $M_2(\mathbb{F}_2)$.



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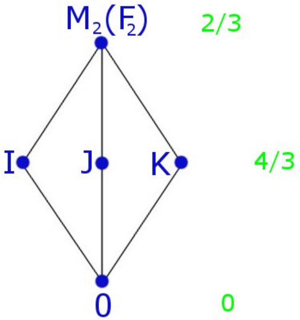

$M_2(\mathbb{F}_2)$ is a two-weight ring. It yields a strongly regular graph with parameters $(16, 6, 2, 2)$.

$M_2(\mathbb{F}_q)$ Construction

Theorem

$M_2(\mathbb{F}_q)$ is a two-weight ring and yields a two-weight code C with

$$w_1 = \frac{q^3 - q^2 - q}{q^3 - q^2 - q + 1} \text{ and } w_2 = \frac{q^2}{q^2 - 1}.$$

$G(C)$ is a strongly regular graph with parameters

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- It was known that $M_2(\mathbb{F}_q)$ yields a strongly regular graph by taking its elements as vertices and joining two matrices if their difference has rank 2. $G(C)$ is isomorphic to this graph.

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- It was known that $M_2(\mathbb{F}_q)$ yields a strongly regular graph by taking its elements as vertices and joining two matrices if their difference has rank 2. $G(C)$ is isomorphic to this graph.
- So we have a new interpretation of the result that the elements of $M_2(\mathbb{F}_q)$ give a strongly regular graph by using the concept of a two-weight code.

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