**Abstract**

# On the minimum pseudoweight of codes with large automorphism group

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Pseudocodewords play a significant role in understanding the performance of binary parity-check codes under linear-programming (LP) or message-passing (MP) decoding. The concept of (AWGNC) pseudoweight of a pseudocodeword was introduced as an analogue to the signal Euclidean distance in the ML decoding scenario. Accordingly, for a binary linear code $\mathcal{C}$ and a parity-check matrix $\boldsymbol{H}$ of $\mathcal{C}$ the minimum pseudoweight $\mathrm{w}^{\min}(\boldsymbol{H})$ may be considered as a first-order measure of error-correcting performance under LP or MP decoding. The minimum pseudoweight is always bounded above by the code's minimum Hamming distance: $\mathrm{w}^{\min}(\boldsymbol{H}) \leq d(\mathcal{C})$. Furthermore, it has been shown that for a random code of a given rate with high probability this inequality is always strict.

In this talk we are interested in codes $\mathcal{C}$ for which equality $\mathrm{w}^{\min}(\boldsymbol{H}) = d(\mathcal{C})$ holds for some parity-check matrix $\boldsymbol{H}$. We show that certain well-known codes $\mathcal{C}$ that have a large automorphism group $\mathrm{Aut}(\mathcal{C})$ belong to this class, including

- the simple parity-check and repetition codes,
- the Hamming and Simplex codes,
- the [8,4,4] extended Hamming code,
- the Euclidean and projective geometry codes (proven also by R. Smarandache and P. O. Vontobel).

The automorphism group of these codes is used in the proofs in different ways. We also report on the progress of proving the conjecture $w^{\min}(\boldsymbol{H}) = d(\mathcal{C})$ for the Golay codes.

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