Abstract

On extremal maximal isotropic codes of Type I-IV

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Let $\mathbb{F}$ be a finite field, and let $C \leq \mathbb{F}^N$ be a linear code. The dual code of $C$ is $C^\perp = \{v \in \mathbb{F}^N \mid \sum_{i=1}^N v_i c_i' = 0 \text{ for all } c \in C\}$, where $J$ is the identity or a field automorphism of order 2. If $C \subseteq C^\perp$ then $C$ is called self-orthogonal and if $C = C^\perp$ then $C$ is called self-dual. Based on a divisibility condition on the Hamming weights $\text{wt}(c) := |\{i \in \{1, \ldots, N\} \mid c_i \neq 0\}|$ of codewords $c \in C$, one basically distinguishes four Types of self-dual codes. Type I codes are, e.g., the self-dual binary codes, in which all weights are automatically divisible by 2. The Hamming weight enumerator $\text{hwe}(C)(x, y) := \sum_{c \in C} x^{N-\text{wt}(c)} y^{\text{wt}(c)} \in \mathbb{C}[x, y]$, a homogeneous polynomial of degree $N$, counts the number of codewords of each weight. Gleason showed that for a Type $T \in \{I, II, III, IV\}$, the weight enumerators of self-dual Type $T$ codes lie in a polynomial ring $\mathbb{C}[p_T, q_T]$, where $p_T$ and $q_T$ themselves are weight enumerators of self-dual Type $T$ codes ([1]).

This very powerful result provides an overview of the possible weight distributions of such codes, and in particular to derive upper bounds on the minimum weight $d(C) := \min_{c \in C} \text{wt}(c)$ (see [4]). The closer the minimum weight comes to this bound, the better the error-correcting capability of the code. A self-dual code is called extremal if its minimum weight reaches the respective upper bound.

Moreover, it follows immediately from Gleason’s Theorem that the length of a self-dual Type $T$ code is always a multiple of $o(T) := \min(\deg(p_T), \deg(q_T)) = \gcd(\deg(p_T), \deg(q_T))$.

In this talk, we consider the case where $N$ is no multiple of $o(T)$. By the above, there exists no self-dual Type $T$ codes for these lengths, but one may still consider maximal self-orthogonal codes. The complex vector space spanned by the dual weight enumerators of these codes is still a module for $\mathbb{C}[p_T, q_T]$, since the orthogonal sum of a self-dual and a maximal self-orthogonal code is again a maximal self-orthogonal code. This module is finitely generated and free ([3]).

Based on the latter observation, one obtains results on the weight distribution similar to those in the case of self-dual codes ([2]). We derive upper bounds on the dual minimum weight $d(C^\perp)$ of self-orthogonal codes of Type I-IV, which gives rise to the notion of dual extremal maximal self-orthogonal codes. Moreover, it is shown that the weight enumerator of a dual extremal code is uniquely determined.
References


