Abstract

On maximal partial spreads of the hermitian variety \( H(3, q^2) \)

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We consider the hermitian variety \( H(3, q^2) \) as the geometry consisting of all totally isotropic subspaces with respect to a given non-singular hermitian form on the projective space \( \text{PG}(3, q^2) \). It consists of points and lines, and it is one of the finite classical generalized quadrangles.

A spread is a set \( L \) of lines of \( H(3, q^2) \) partitioning the point set of \( H(3, q^2) \). It is known for a long time that no spreads of \( H(3, q^2) \) exist. A partial spread is a set \( L \) of lines of \( H(3, q^2) \) such that every point of \( H(3, q^2) \) is contained in at most one element of \( L \). A partial spread is called maximal if it is not a proper subset of any (partial) spread. The natural question is how large a maximal partial spread of \( H(3, q^2) \) can be.

We discuss the currently best known upper bound on the size of maximal partial spreads of \( H(3, q^2) \). This upper bound is sharp for \( q = 2 \) and \( q = 3 \), but probably not for all \( q > 3 \). Computer searches confirm this for \( q = 4 \) and \( q = 5 \). We discuss known examples of large maximal partial spreads of \( H(3, q^2) \).