Primitive central idempotents of finite group rings of symmetric and alternating groups in characteristic 2

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1 The primitive central idempotents of group rings of symmetric groups in characteristic 2

It is well known that the conjugacy classes of S_n can be indexed by the partitions of n. We write $\mu = 1^{\alpha_1}, ..., n^{\alpha_n}$ for the partition

$$\mu = (\underbrace{1, ..., 1}_{\alpha_1}, \underbrace{2, ..., 2}_{\alpha_2}, ...)$$

of n. We define

$$W(\mu) := \sum_{i=2}^{n} i \cdot \alpha_i$$

and call it the *essential weight* of the partition μ. For our purpose it is convenient to ignore the parts equal to 1 in the partition because an element like $(1, 2, 3) \in S_3$ is also an element of bigger symmetric groups. So we write $\mu = 2^{\alpha_2}, ..., n^{\alpha_n}$ for a partition and the corresponding class C_μ is a class of an arbitrary symmetric group S_n with $n \geq W(\mu)$ depending on the context, i.e. C_2 denotes the conjugacy class of transpositions in every symmetric group S_n, $n \geq 2$. If $\mu = 2^{\alpha_2}, ..., n^{\alpha_n}$ is a partition we write $\overline{2^{\alpha_2}, ..., n^{\alpha_n}}$ for the class sum $C_\mu^+ \in \mathbb{F}_2 S_m$, where $m \geq W(\mu)$.

According to Theorem 1 of [1] one can easily deduce the primitive central idempotents of $\mathbb{F}_2 S_n$ for $n < 54$ from the primitive central idempotents of $\mathbb{F}_2 S_{54}$ and $\mathbb{F}_2 S_{53}$. To simplify that task we added tokens of the form $|_{16}$ to indicate where the primitive central idempotent of $\mathbb{F}_2 S_{16}$ ends.

Primitive central idempotents of $\mathbb{F}_2 S_n$ for n odd and $n \leq 53$:

$$\begin{aligned}
e_1 = {} & \overline{1}|_1 + \overline{3}|_3 + \overline{5}|_5 + \overline{3,5} + \overline{9}|_9 + \overline{7,9} + \overline{5,11} + \overline{3,13} + \overline{17}|_{17} + \overline{3,7,11}|_{21} + \overline{5,7,11} + \overline{3,9,11} + \overline{3,7,13}|_{23} + \overline{5,9,11} + \overline{5,7,13} + \\
& \overline{3,9,13}|_{25} + \overline{3,5,7,11} + \overline{7,9,11} + \overline{5,9,13} + \overline{3,11,13} + \overline{3,7,17}|_{27} + \overline{3,5,9,11} + \overline{3,5,7,13} + \overline{7,9,13} + \overline{5,11,13} + \overline{5,7,17} + \\
& \overline{3,9,17}|_{29} + \overline{3,5,9,13} + \overline{5,9,17}|_{31} + \overline{5,7,9,11} + \overline{3,5,11,13} + \overline{3,5,7,17} + \overline{15,17} + \overline{13,19} + \overline{11,21} + \overline{9,23} + \overline{7,25} + \overline{5,27} + \overline{3,29} + \\
& \overline{9,11,13} + \overline{7,9,17} + \overline{3,13,17} + \overline{33}|_{33} + \overline{5,7,9,13} + \overline{3,7,11,13} + \overline{3,5,9,17} + \overline{3,5,7,9,11} + \overline{7,11,17} + \overline{5,13,17} + \overline{3,11,21} + \\
& \overline{3,7,25}|_{35} + \overline{5,7,11,13} + \overline{3,9,11,13} + \overline{3,5,7,9,13} + \overline{9,11,17} + \overline{7,13,17} + \overline{3,15,19} + \overline{5,11,21} + \overline{3,13,21} + \overline{3,11,23} + \\
& \overline{5,7,25} + \overline{3,9,25} + \overline{3,7,27}|_{37} + \overline{5,7,9,17} + \overline{3,5,13,17} + \overline{5,15,19} + \overline{3,17,19} + \overline{5,13,21} + \overline{3,15,21} + \overline{5,11,23} + \overline{3,13,23} + \\
& \overline{5,9,25} + \overline{3,11,25} + \overline{5,7,27} + \overline{3,9,27} + \overline{3,7,29}|_{39} + \overline{7,9,11,13} + \overline{5,7,11,17} + \overline{3,7,13,17} + \overline{3,5,11,21} + \overline{3,5,7,25} + \\
& \overline{3,5,9,11,13} + \overline{3,5,7,9,17} + \overline{11,13,17} + \overline{5,17,19} + \overline{9,11,21} + \overline{5,15,21} + \overline{5,13,23} + \overline{7,9,25} + \overline{5,11,25} + \overline{5,9,27} + \overline{5,7,29} + \\
& \overline{3,9,29}|_{41} + \overline{5,9,11,17} + \overline{3,9,13,17} + \overline{3,5,15,19} + \overline{3,5,13,21} + \overline{3,5,11,23} + \overline{3,5,9,25} + \overline{3,5,7,27} + \overline{9,15,19} + \overline{9,13,21} + \\
& \overline{3,19,21} + \overline{9,11,23} + \overline{3,17,23} + \overline{3,15,25} + \overline{7,9,27} + \overline{3,13,27} + \overline{5,9,29} + \overline{3,11,29} + \overline{3,7,33}|_{43} + \overline{3,5,17,19} + \overline{3,5,15,21} + \\
& \overline{3,5,13,23} + \overline{3,5,11,25} + \overline{3,5,9,27} + \overline{3,5,7,29} + \overline{9,17,19} + \overline{9,15,21} + \overline{5,19,21} + \overline{9,13,23} + \overline{5,17,23} + \overline{9,11,25} + \overline{5,15,25} + \\
& \overline{5,13,27} + \overline{7,9,29} + \overline{5,11,29} + \overline{5,7,33} + \overline{3,9,33}|_{45} + \overline{5,9,15,17} + \overline{5,9,13,19} + \overline{5,9,11,21} + \overline{5,7,9,25} + \overline{3,5,9,29} + \\
& \overline{5,9,33}|_{47} + \overline{3,5,7,9,11,13} + \overline{7,9,15,17} + \overline{5,11,15,17} + \overline{7,9,13,19} + \overline{5,11,13,19} + \overline{3,11,13,21} + \overline{3,5,19,21} + \overline{5,9,11,23} + \\
& \overline{3,5,17,23} + \overline{3,7,13,25} + \overline{3,5,15,25} + \overline{5,7,9,27} + \overline{3,5,13,27} + \overline{3,5,11,29} + \overline{3,5,7,33} + \overline{5,7,9,11,17} + \overline{3,7,9,13,17} + \\
& \overline{3,5,11,13,17} + \overline{3,5,9,15,17} + \overline{3,5,9,13,19} + \overline{3,5,9,11,21} + \overline{11,17,21} + \overline{9,19,21} + \overline{9,17,23} + \overline{3,5,7,9,25} + \overline{9,15,25} + \\
& \overline{7,17,25} + \overline{9,13,27} + \overline{9,11,29} + \overline{3,17,29} + \overline{7,9,33} + \overline{3,13,33}|_{49} + \overline{7,11,15,17} + \overline{5,13,15,17} + \overline{7,11,13,19} + \overline{7,9,15,19} + \\
& \overline{5,11,15,19} + \overline{3,13,15,19} + \overline{7,9,13,21} + \overline{5,9,13,23} + \overline{3,11,13,23} + \overline{5,9,11,25} + \overline{5,7,13,25} + \overline{3,9,13,25} + \overline{3,7,13,27} + \\
& \overline{5,7,9,29} + \overline{3,7,11,29} + \overline{3,5,9,33} + \overline{3,7,9,15,17} + \overline{3,5,11,15,17} + \overline{3,7,9,13,19} + \overline{3,5,11,13,19} + \overline{15,17,19} + \\
& \overline{3,7,9,11,21} + \overline{13,17,21} + \overline{3,5,9,11,23} + \overline{11,17,23} + \overline{3,5,7,11,25} + \overline{9,17,25} + \overline{3,23,25} + \overline{3,5,7,9,27} + \overline{7,17,27} + \\
& \overline{3,21,27} + \overline{5,17,29} + \overline{3,19,29} + \overline{7,11,33} + \overline{5,13,33} + \overline{3,15,33}|_{51} + \overline{9,11,15,17} + \overline{7,13,15,17} + \overline{9,11,13,19} + \overline{7,9,17,19} + \\
& \overline{5,11,17,19} + \overline{3,13,17,19} + \overline{7,11,13,21} + \overline{7,9,15,21} + \overline{5,11,15,21} + \overline{3,13,15,21} + \overline{5,11,13,23} + \overline{3,11,13,25} + \\
& \overline{5,7,13,27} + \overline{3,9,13,27} + \overline{5,7,11,29} + \overline{3,9,11,29} + \overline{5,7,9,15,17} + \overline{3,5,13,15,17} + \overline{5,7,9,13,19} + \overline{5,7,9,11,21} + \\
& \overline{3,5,11,13,21} + \overline{15,17,21} + \overline{3,5,9,13,23} + \overline{13,17,23} + \overline{3,5,7,13,25} + \overline{11,17,25} + \overline{5,23,25} + \overline{9,17,27} + \overline{5,21,27} + \\
& \overline{3,5,7,9,29} + \overline{7,17,29} + \overline{5,19,29} + \overline{9,11,33} + \overline{7,13,33} + \overline{5,15,33} + \overline{3,17,33}
\end{aligned}$$

$$
\begin{aligned}
e_2 \quad = \quad & \overline{3}|_{3}+\overline{5}|_{5}+\overline{3,5}+\overline{9}|_{9}+\overline{5,9}|_{15}+\overline{3,13}+\overline{3,5,9}+\overline{17}|_{17}+\overline{7,11}+\overline{5,13}+\overline{3,7,9}+\overline{3,5,11}|_{19}+\overline{9,11}+\overline{7,13}+\overline{5,7,9}+\overline{3,7,11}+ \\
& \overline{3,5,13}|_{21}+\overline{5,17}+\overline{5,7,11}+\overline{3,9,11}+\overline{3,7,13}|_{23}+\overline{3,5,7,9}+\overline{11,13}+\overline{7,17}+\overline{5,7,13}+\overline{3,5,17}|_{25}+\overline{3,5,7,11}+\overline{9,17}+ \\
& \overline{7,9,11}+\overline{5,9,13}+\overline{3,11,13}+\overline{3,7,17}|_{27}+\overline{3,5,9,11}+\overline{3,5,7,13}+\overline{7,9,13}+\overline{5,11,13}+\overline{5,7,17}+\overline{3,9,17}|_{29}+\overline{3,5,9,13}+ \\
& \overline{5,9,17}|_{31}+\overline{3,7,9,13}+\overline{3,5,7,17}+\overline{11,21}+\overline{7,25}+\overline{3,29}+\overline{9,11,13}+\overline{5,11,17}+\overline{33}|_{33}+\overline{5,7,9,13}+\overline{3,7,11,13}+\overline{3,5,9,17}+ \\
& \overline{15,19}+\overline{13,21}+\overline{11,23}+\overline{9,25}+\overline{7,27}+\overline{5,29}+\overline{3,5,7,9,11}+\overline{7,11,17}+\overline{5,13,17}+\overline{3,15,17}+\overline{3,13,19}+\overline{3,9,23}+ \\
& \overline{3,5,27}|_{35}+\overline{5,7,11,13}+\overline{3,9,11,13}+\overline{17,19}+\overline{15,21}+\overline{13,23}+\overline{11,25}+\overline{9,27}+\overline{7,29}+\overline{3,5,7,9,13}+\overline{9,11,17}+\overline{7,13,17}+ \\
& \overline{5,15,17}+\overline{5,13,19}+\overline{3,15,19}+\overline{3,13,21}+\overline{5,9,23}+\overline{3,11,23}+\overline{3,9,25}+\overline{3,7,27}+\overline{3,5,29}|_{37}+\overline{5,7,9,17}+\overline{3,5,13,17}+ \\
& \overline{5,33}+\overline{5,15,19}+\overline{3,17,19}+\overline{5,13,21}+\overline{3,15,21}+\overline{5,11,23}+\overline{3,13,23}+\overline{5,9,25}+\overline{3,11,25}+\overline{5,7,27}+\overline{3,9,27}+\overline{3,7,29}|_{39}+ \\
& \overline{7,9,11,13}+\overline{5,7,11,17}+\overline{3,7,13,17}+\overline{3,5,15,17}+\overline{3,5,13,19}+\overline{19,21}+\overline{3,5,9,23}+\overline{17,23}+\overline{15,25}+\overline{13,27}+\overline{11,29}+ \\
& \overline{7,33}+\overline{3,5,9,11,13}+\overline{3,5,7,9,17}+\overline{11,13,17}+\overline{9,15,17}+\overline{9,13,19}+\overline{5,17,19}+\overline{5,15,21}+\overline{5,13,23}+\overline{5,11,25}+\overline{5,7,29}+ \\
& \overline{3,5,33}|_{41}+\overline{5,9,11,17}+\overline{3,9,13,17}+\overline{3,5,15,19}+\overline{3,5,13,21}+\overline{3,5,11,23}+\overline{3,5,9,25}+\overline{3,5,7,27}+\overline{9,33}+\overline{9,15,19}+ \\
& \overline{9,13,21}+\overline{3,19,21}+\overline{9,11,23}+\overline{3,17,23}+\overline{3,15,25}+\overline{7,9,27}+\overline{3,13,27}+\overline{5,9,29}+\overline{3,11,29}+\overline{3,7,33}|_{43}+\overline{3,5,17,19}+ \\
& \overline{3,5,15,21}+\overline{3,5,13,23}+\overline{3,5,11,25}+\overline{3,5,9,27}+\overline{3,5,7,29}+\overline{9,17,19}+\overline{9,15,21}+\overline{5,19,21}+\overline{9,13,23}+\overline{5,17,23}+ \\
& \overline{9,11,25}+\overline{5,15,25}+\overline{5,13,27}+\overline{7,9,29}+\overline{5,11,29}+\overline{5,7,33}+\overline{3,9,33}|_{45}+\overline{5,9,15,17}+\overline{5,9,13,19}+\overline{5,9,11,21}+ \\
& \overline{5,7,9,25}+\overline{3,5,9,29}+\overline{5,9,33}|_{47}+\overline{3,5,7,9,11,13}+\overline{3,13,15,17}+\overline{7,9,11,21}+\overline{3,5,19,21}+\overline{3,9,13,23}+\overline{3,5,17,23}+ \\
& \overline{5,7,11,25}+\overline{3,5,15,25}+\overline{23,25}+\overline{21,27}+\overline{3,7,9,29}+\overline{19,29}+\overline{3,5,7,33}+\overline{15,33}+\overline{5,7,9,11,17}+\overline{3,7,9,13,17}+ \\
& \overline{3,5,11,13,17}+\overline{3,5,9,15,17}+\overline{3,5,9,13,19}+\overline{13,17,19}+\overline{3,5,9,11,21}+\overline{9,19,21}+\overline{3,5,7,9,25}+\overline{9,15,25}+\overline{9,13,27}+ \\
& \overline{5,17,27}+\overline{9,11,29}+\overline{5,11,33}|_{49}+\overline{7,11,15,17}+\overline{5,13,15,17}+\overline{7,11,13,19}+\overline{7,9,15,19}+\overline{5,11,15,19}+\overline{3,13,15,19}+ \\
& \overline{7,9,13,21}+\overline{5,9,13,23}+\overline{3,11,13,23}+\overline{5,9,11,25}+\overline{5,7,13,25}+\overline{3,9,13,25}+\overline{3,7,13,27}+\overline{5,7,9,29}+\overline{3,7,11,29}+ \\
& \overline{3,5,9,33}+\overline{17,33}+\overline{3,7,9,15,17}+\overline{3,5,11,15,17}+\overline{3,7,9,13,19}+\overline{3,5,11,13,19}+\overline{15,17,19}+\overline{3,7,9,11,21}+\overline{13,17,21}+ \\
& \overline{3,5,9,11,23}+\overline{11,17,23}+\overline{3,5,7,11,25}+\overline{9,17,25}+\overline{3,23,25}+\overline{3,5,7,9,27}+\overline{7,17,27}+\overline{3,21,27}+\overline{5,17,29}+\overline{3,19,29}+ \\
& \overline{7,11,33}+\overline{5,13,33}+\overline{3,15,33}|_{51}+\overline{9,11,15,17}+\overline{7,13,15,17}+\overline{9,11,13,19}+\overline{7,9,17,19}+\overline{5,11,17,19}+\overline{3,13,17,19}+ \\
& \overline{7,11,13,21}+\overline{7,9,15,21}+\overline{5,11,15,21}+\overline{3,13,15,21}+\overline{5,11,13,23}+\overline{3,11,13,25}+\overline{5,7,13,27}+\overline{3,9,13,27}+\overline{5,7,11,29}+ \\
& \overline{3,9,11,29}+\overline{5,7,9,15,17}+\overline{3,5,13,15,17}+\overline{5,7,9,13,19}+\overline{5,7,9,11,21}+\overline{3,5,11,13,21}+\overline{15,17,21}+\overline{3,5,9,13,23}+ \\
& \overline{13,17,23}+\overline{3,5,7,13,25}+\overline{11,17,25}+\overline{5,23,25}+\overline{9,17,27}+\overline{5,21,27}+\overline{3,5,7,9,29}+\overline{7,17,29}+\overline{5,19,29}+\overline{9,11,33}+ \\
& \overline{7,13,33}+\overline{5,15,33}+\overline{3,17,33}
\end{aligned}
$$

$$
\begin{aligned}
e_3 \quad = \quad & \overline{5,9}|_{15}+\overline{7,9}+\overline{5,11}+\overline{3,5,9}|_{17}+\overline{7,11}+\overline{5,13}+\overline{3,7,9}+\overline{3,5,11}|_{19}+\overline{9,11}+\overline{7,13}+\overline{5,7,9}+\overline{3,7,11}+\overline{3,5,13}|_{21}+\overline{5,17}+ \\
& \overline{5,7,11}+\overline{3,9,11}+\overline{3,7,13}|_{23}+\overline{3,5,7,9}+\overline{11,13}+\overline{7,17}+\overline{5,7,13}+\overline{3,5,17}|_{25}+\overline{3,5,7,11}+\overline{9,17}+\overline{7,9,11}+\overline{5,9,13}+ \\
& \overline{3,11,13}+\overline{3,7,17}|_{27}+\overline{3,5,9,11}+\overline{3,5,7,13}+\overline{7,9,13}+\overline{5,11,13}+\overline{5,7,17}+\overline{3,9,17}|_{29}+\overline{3,5,9,13}+\overline{5,9,17}|_{31}+\overline{3,7,9,13}+ \\
& \overline{3,5,7,17}+\overline{15,17}+\overline{13,19}+\overline{9,23}+\overline{5,27}+\overline{9,11,13}+\overline{5,11,17}|_{33}+\overline{5,7,9,13}+\overline{3,7,11,13}+\overline{3,5,9,17}+\overline{15,19}+\overline{13,21}+ \\
& \overline{11,23}+\overline{9,25}+\overline{7,27}+\overline{5,29}+\overline{3,5,7,9,11}+\overline{7,11,17}+\overline{5,13,17}+\overline{3,15,17}+\overline{3,13,19}+\overline{3,9,23}+\overline{3,5,27}|_{35}+\overline{5,7,11,13}+ \\
& \overline{3,9,11,13}+\overline{17,19}+\overline{15,21}+\overline{13,23}+\overline{11,25}+\overline{9,27}+\overline{7,29}+\overline{3,5,7,9,13}+\overline{9,11,17}+\overline{7,13,17}+\overline{5,15,17}+\overline{5,13,19}+ \\
& \overline{3,15,19}+\overline{3,13,21}+\overline{5,9,23}+\overline{3,11,23}+\overline{3,9,25}+\overline{3,7,27}+\overline{3,5,29}|_{37}+\overline{5,7,9,17}+\overline{3,5,13,17}+\overline{5,33}+\overline{5,15,19}+\overline{3,17,19}+ \\
& \overline{5,13,21}+\overline{3,15,21}+\overline{5,11,23}+\overline{3,13,23}+\overline{5,9,25}+\overline{3,11,25}+\overline{5,7,27}+\overline{3,9,27}+\overline{3,7,29}|_{39}+\overline{7,9,11,13}+\overline{5,7,11,17}+ \\
& \overline{3,7,13,17}+\overline{3,5,15,17}+\overline{3,5,13,19}+\overline{19,21}+\overline{3,5,9,23}+\overline{17,23}+\overline{15,25}+\overline{13,27}+\overline{11,29}+\overline{7,33}+\overline{3,5,9,11,13}+ \\
& \overline{3,5,7,9,17}+\overline{11,13,17}+\overline{9,15,17}+\overline{9,13,19}+\overline{5,17,19}+\overline{5,15,21}+\overline{5,13,23}+\overline{5,11,25}+\overline{5,7,29}+\overline{3,5,33}|_{41}+ \\
& \overline{5,9,11,17}+\overline{3,9,13,17}+\overline{3,5,15,19}+\overline{3,5,13,21}+\overline{3,5,11,23}+\overline{3,5,9,25}+\overline{3,5,7,27}+\overline{9,33}+\overline{9,15,19}+\overline{9,13,21}+ \\
& \overline{3,19,21}+\overline{9,11,23}+\overline{3,17,23}+\overline{3,15,25}+\overline{7,9,27}+\overline{3,13,27}+\overline{5,9,29}+\overline{3,11,29}+\overline{3,7,33}|_{43}+\overline{3,5,17,19}+\overline{3,5,15,21}+ \\
& \overline{3,5,13,23}+\overline{3,5,11,25}+\overline{3,5,9,27}+\overline{3,5,7,29}+\overline{9,17,19}+\overline{9,15,21}+\overline{5,19,21}+\overline{9,13,23}+\overline{5,17,23}+\overline{9,11,25}+\overline{5,15,25}+ \\
& \overline{5,13,27}+\overline{7,9,29}+\overline{5,11,29}+\overline{5,7,33}+\overline{3,9,33}|_{45}+\overline{5,9,15,17}+\overline{5,9,13,19}+\overline{5,9,11,21}+\overline{5,7,9,25}+\overline{3,5,9,29}+ \\
& \overline{5,9,33}|_{47}+\overline{3,5,7,9,11,13}+\overline{3,13,15,17}+\overline{7,9,11,21}+\overline{3,5,19,21}+\overline{3,9,13,23}+\overline{3,5,17,23}+\overline{5,7,11,25}+\overline{3,5,15,25}+ \\
& \overline{23,25}+\overline{21,27}+\overline{3,7,9,29}+\overline{19,29}+\overline{3,5,7,33}+\overline{15,33}+\overline{5,7,9,11,17}+\overline{3,7,9,13,17}+\overline{3,5,11,13,17}+\overline{3,5,9,15,17}+ \\
& \overline{3,5,9,13,19}+\overline{13,17,19}+\overline{3,5,9,11,21}+\overline{9,19,21}+\overline{3,5,7,9,25}+\overline{9,15,25}+\overline{9,13,27}+\overline{5,17,27}+\overline{9,11,29}+\overline{5,11,33}|_{49}+ \\
& \overline{7,11,15,17}+\overline{5,13,15,17}+\overline{7,11,13,19}+\overline{7,9,15,19}+\overline{5,11,15,19}+\overline{3,13,15,19}+\overline{7,9,13,21}+\overline{5,9,13,23}+\overline{3,11,13,23}+ \\
& \overline{5,9,11,25}+\overline{5,7,13,25}+\overline{3,9,13,25}+\overline{3,7,13,27}+\overline{5,7,9,29}+\overline{3,7,11,29}+\overline{3,5,9,33}+\overline{17,33}+\overline{3,7,9,15,17}+ \\
& \overline{3,5,11,15,17}+\overline{3,7,9,13,19}+\overline{3,5,11,13,19}+\overline{15,17,19}+\overline{3,7,9,11,21}+\overline{13,17,21}+\overline{3,5,9,11,23}+\overline{11,17,23}+ \\
& \overline{3,5,7,11,25}+\overline{9,17,25}+\overline{3,23,25}+\overline{3,5,7,9,27}+\overline{7,17,27}+\overline{3,21,27}+\overline{5,17,29}+\overline{3,19,29}+\overline{7,11,33}+\overline{5,13,33}+ \\
& \overline{3,15,33}|_{51}+\overline{9,11,15,17}+\overline{7,13,15,17}+\overline{9,11,13,19}+\overline{7,9,17,19}+\overline{5,11,17,19}+\overline{3,13,17,19}+\overline{7,11,13,21}+\overline{7,9,15,21}+ \\
& \overline{5,11,15,21}+\overline{3,13,15,21}+\overline{5,11,13,23}+\overline{3,11,13,25}+\overline{5,7,13,27}+\overline{3,9,13,27}+\overline{5,7,11,29}+\overline{3,9,11,29}+\overline{5,7,9,15,17}+ \\
& \overline{3,5,13,15,17}+\overline{5,7,9,13,19}+\overline{5,7,9,11,21}+\overline{3,5,11,13,21}+\overline{15,17,21}+\overline{3,5,9,13,23}+\overline{13,17,23}+\overline{3,5,7,13,25}+ \\
& \overline{11,17,25}+\overline{5,23,25}+\overline{9,17,27}+\overline{5,21,27}+\overline{3,5,7,9,29}+\overline{7,17,29}+\overline{5,19,29}+\overline{9,11,33}+\overline{7,13,33}+\overline{5,15,33}+\overline{3,17,33}
\end{aligned}
$$

$$
\begin{aligned}
e_4 \quad = \quad & \overline{3,7,11}|_{21}+\overline{5,7,11}+\overline{3,9,11}+\overline{3,7,13}|_{23}+\overline{5,9,11}+\overline{5,7,13}+\overline{3,9,13}|_{25}+\overline{3,5,7,11}+\overline{7,9,11}+\overline{5,9,13}+\overline{3,11,13}+ \\
& \overline{3,7,17}|_{27}+\overline{3,5,9,11}+\overline{3,5,7,13}+\overline{7,9,13}+\overline{5,11,13}+\overline{5,7,17}+\overline{3,9,17}|_{29}+\overline{3,5,9,13}+\overline{5,9,17}|_{31}+\overline{5,7,9,11}+ \\
& \overline{3,5,11,13}+\overline{3,5,7,17}+\overline{9,11,13}+\overline{7,9,17}+\overline{3,13,17}|_{33}+\overline{5,7,9,13}+\overline{3,7,11,13}+\overline{3,5,9,17}+\overline{3,5,7,9,11}+\overline{7,11,17}+ \\
& \overline{5,13,17}+\overline{3,11,21}+\overline{3,7,25}|_{35}+\overline{5,7,11,13}+\overline{3,9,11,13}+\overline{3,5,7,9,13}+\overline{9,11,17}+\overline{7,13,17}+\overline{3,15,19}+\overline{5,11,21}+ \\
& \overline{3,13,21}+\overline{3,11,23}+\overline{5,7,25}+\overline{3,9,25}+\overline{3,7,27}|_{37}+\overline{5,7,9,17}+\overline{3,5,13,17}+\overline{5,15,19}+\overline{3,17,19}+\overline{5,13,21}+\overline{3,15,21}+ \\
& \overline{5,11,23}+\overline{3,13,23}+\overline{5,9,25}+\overline{3,11,25}+\overline{5,7,27}+\overline{3,9,27}+\overline{3,7,29}|_{39}+\overline{7,9,11,13}+\overline{5,7,11,17}+\overline{3,7,13,17}+ \\
& \overline{3,5,11,21}+\overline{3,5,7,25}+\overline{3,5,9,11,13}+\overline{3,5,7,9,17}+\overline{11,13,17}+\overline{5,17,19}+\overline{9,11,21}+\overline{5,15,21}+\overline{5,13,23}+\overline{7,9,25}+ \\
& \overline{5,11,25}+\overline{5,9,27}+\overline{5,7,29}+\overline{3,9,29}|_{41}+\overline{5,9,11,17}+\overline{3,9,13,17}+\overline{3,5,15,19}+\overline{3,5,13,21}+\overline{3,5,11,23}+\overline{3,5,9,25}+ \\
& \overline{3,5,7,27}+\overline{9,15,19}+\overline{9,13,21}+\overline{3,19,21}+\overline{9,11,23}+\overline{3,17,23}+\overline{3,15,25}+\overline{7,9,27}+\overline{3,13,27}+\overline{5,9,29}+\overline{3,11,29}+ \\
& \overline{3,7,33}|_{43}+\overline{5,9,13,17}+\overline{3,5,17,19}+\overline{3,5,15,21}+\overline{3,5,13,23}+\overline{3,5,11,25}+\overline{3,5,9,27}+\overline{3,5,7,29}+\overline{9,17,19}+\overline{9,15,21}+ \\
& \overline{5,19,21}+\overline{9,13,23}+\overline{5,17,23}+\overline{9,11,25}+\overline{5,15,25}+\overline{5,13,27}+\overline{7,9,29}+\overline{5,11,29}+\overline{5,7,33}+\overline{3,9,33}|_{45}+\overline{7,9,13,17}+ \\
& \overline{5,11,13,17}+\overline{5,9,11,21}+\overline{5,7,9,25}+\overline{3,5,9,29}+\overline{3,5,9,13,17}+\overline{5,9,33}|_{47}+\overline{3,5,7,9,11,13}+\overline{7,11,13,17}+\overline{5,9,15,19}+ \\
& \overline{5,9,13,21}+\overline{3,11,13,21}+\overline{3,5,19,21}+\overline{5,9,11,23}+\overline{3,5,17,23}+\overline{3,7,13,25}+\overline{3,5,15,25}+\overline{5,7,9,27}+\overline{3,5,13,27}+ \\
& \overline{3,5,11,29}+\overline{3,5,7,33}+\overline{5,7,9,11,17}+\overline{3,5,9,11,21}+\overline{11,17,21}+\overline{9,19,21}+\overline{9,17,23}+\overline{3,5,7,9,25}+\overline{9,15,25}+\overline{7,17,25}+ \\
& \overline{9,13,27}+\overline{9,11,29}+\overline{3,17,29}+\overline{7,9,33}+\overline{3,13,33}|_{49}+\overline{9,11,13,17}+\overline{3,13,15,19}+\overline{5,9,17,19}+\overline{5,11,13,21}+\overline{5,9,15,21}+ \\
& \overline{5,9,13,23}+\overline{3,11,13,23}+\overline{5,9,11,25}+\overline{5,7,13,25}+\overline{3,9,13,25}+\overline{3,7,13,27}+\overline{5,7,9,29}+\overline{3,7,11,29}+\overline{3,5,9,33}+ \\
& \overline{5,7,9,13,17}+\overline{3,7,11,13,17}+\overline{3,5,9,15,19}+\overline{15,17,19}+\overline{3,7,9,11,21}+\overline{3,5,9,13,21}+\overline{13,17,21}+\overline{3,5,9,11,23}+ \\
& \overline{11,17,23}+\overline{3,5,7,11,25}+\overline{9,17,25}+\overline{3,23,25}+\overline{3,5,7,9,27}+\overline{7,17,27}+\overline{3,21,27}+\overline{5,17,29}+\overline{3,19,29}+\overline{7,11,33}+ \\
& \overline{5,13,33}+\overline{3,15,33}|_{51}+\overline{7,11,15,19}+\overline{5,13,15,19}+\overline{3,13,17,19}+\overline{3,13,15,21}+\overline{5,11,13,23}+\overline{5,9,13,25}+\overline{3,11,13,25}+ \\
& \overline{5,7,13,27}+\overline{3,9,13,27}+\overline{5,7,11,29}+\overline{3,9,11,29}+\overline{5,7,11,13,17}+\overline{3,9,11,13,17}+\overline{3,7,11,15,17}+\overline{3,7,11,13,19}+ \\
& \overline{3,7,9,15,19}+\overline{3,5,11,15,19}+\overline{3,5,9,17,19}+\overline{5,7,9,11,21}+\overline{3,7,9,13,21}+\overline{3,5,9,15,21}+\overline{15,17,21}+\overline{3,5,9,13,23}+ \\
& \overline{13,17,23}+\overline{3,5,7,13,25}+\overline{11,17,25}+\overline{5,23,25}+\overline{9,17,27}+\overline{5,21,27}+\overline{3,5,7,9,29}+\overline{7,17,29}+\overline{5,19,29}+\overline{9,11,33}+ \\
& \overline{7,13,33}+\overline{5,15,33}+\overline{3,17,33}
\end{aligned}
$$

$$
\begin{aligned}
e_5 \quad = \quad & \overline{5,9,13,17}|_{45} + \overline{7,9,13,17} + \overline{5,11,13,17} + \overline{5,9,15,17} + \overline{5,9,13,19} + \overline{3,5,9,13,17}|_{47} + \overline{7,11,13,17} + \overline{7,9,15,17} + \\
& \overline{5,11,15,17} + \overline{7,9,13,19} + \overline{5,11,13,19} + \overline{5,9,15,19} + \overline{5,9,13,21} + \overline{3,7,9,13,17} + \overline{3,5,11,13,17} + \overline{3,5,9,15,17} + \\
& \overline{3,5,9,13,19}|_{49} + \overline{9,11,13,17} + \overline{7,11,15,17} + \overline{5,13,15,17} + \overline{7,11,13,19} + \overline{7,9,15,19} + \overline{5,11,15,19} + \overline{5,9,17,19} + \\
& \overline{7,9,13,21} + \overline{5,11,13,21} + \overline{5,9,15,21} + \overline{5,7,9,13,17} + \overline{3,7,11,13,17} + \overline{3,7,9,15,17} + \overline{3,5,11,15,17} + \overline{3,7,9,13,19} + \\
& \overline{3,5,11,13,19} + \overline{3,5,9,15,19} + \overline{3,5,9,13,21}|_{51} + \overline{9,11,15,17} + \overline{7,13,15,17} + \overline{9,11,13,19} + \overline{7,11,15,19} + \overline{5,13,15,19} + \\
& \overline{7,9,17,19} + \overline{5,11,17,19} + \overline{7,11,13,21} + \overline{7,9,15,21} + \overline{5,11,15,21} + \overline{5,9,13,25} + \overline{5,7,11,13,17} + \overline{3,9,11,13,17} + \\
& \overline{5,7,9,15,17} + \overline{3,7,11,15,17} + \overline{3,5,13,15,17} + \overline{5,7,9,13,19} + \overline{3,7,11,13,19} + \overline{3,7,9,15,19} + \overline{3,5,11,15,19} + \\
& \overline{3,5,9,17,19} + \overline{3,7,9,13,21} + \overline{3,5,11,13,21} + \overline{3,5,9,15,21}
\end{aligned}
$$

Primitive central idempotents of $\mathbb{F}_2S_n$ for n even and $n \leq 54$:

$$
\begin{aligned}
e_1 \quad = \quad & \overline{1}|_2 + \overline{5}|_6 + \overline{7} + \overline{3,5}|_8 + \overline{9}|_{10} + \overline{15} + \overline{7,9} + \overline{5,11} + \overline{3,13}|_{16} + \overline{17}|_{18} + \overline{5,9,13}|_{28} + \overline{7,9,13} + \overline{5,11,13} + \overline{5,9,15} + \\
& \overline{3,5,9,13}|_{30} + \overline{7,11,13} + \overline{7,9,15} + \overline{5,11,15} + \overline{5,9,17} + \overline{31} + \overline{3,7,9,13} + \overline{3,5,11,13} + \overline{3,5,9,15} + \overline{15,17} + \overline{13,19} + \\
& \overline{11,21} + \overline{9,23} + \overline{7,25} + \overline{5,27} + \overline{3,29}|_{32} + \overline{9,11,13} + \overline{7,11,15} + \overline{5,13,15} + \overline{7,9,17} + \overline{5,11,17} + \overline{33} + \overline{5,7,9,13} + \overline{3,7,11,13} + \\
& \overline{3,7,9,15} + \overline{3,5,11,15} + \overline{3,5,9,17}|_{34} + \overline{9,11,15} + \overline{7,13,15} + \overline{7,11,17} + \overline{5,13,17} + \overline{5,7,11,13} + \overline{3,9,11,13} + \overline{5,7,9,15} + \\
& \overline{3,5,13,15} + \overline{3,7,9,17} + \overline{3,5,11,17}|_{36} + \overline{3,5,7,9,13} + \overline{9,11,17} + \overline{7,13,17} + \overline{5,13,19} + \overline{5,9,23} + \overline{5,7,9,17} + \overline{3,5,13,17}|_{38} + \\
& \overline{3,5,7,11,13} + \overline{3,5,7,9,15} + \overline{11,13,15} + \overline{7,13,19} + \overline{5,15,19} + \overline{5,13,21} + \overline{7,9,23} + \overline{5,11,23} + \overline{5,9,25} + \overline{7,9,11,13} + \\
& \overline{5,9,11,15} + \overline{3,9,13,15} + \overline{3,5,13,19} + \overline{3,5,9,23}|_{40} + \overline{3,5,9,11,13} + \overline{3,5,7,9,17} + \overline{11,13,17} + \overline{9,13,19} + \overline{7,15,19} + \\
& \overline{5,17,19} + \overline{7,13,21} + \overline{5,15,21} + \overline{7,11,23} + \overline{5,13,23} + \overline{7,9,25} + \overline{5,11,25} + \overline{5,9,11,17} + \overline{3,9,13,17} + \overline{3,7,15,17} + \\
& \overline{3,7,13,19} + \overline{3,5,15,19} + \overline{3,7,11,21} + \overline{3,5,13,21} + \overline{3,7,9,23} + \overline{3,5,11,23} + \overline{3,5,9,25}|_{42} + \overline{9,15,19} + \overline{7,17,19} + \overline{9,13,21} + \\
& \overline{7,15,21} + \overline{9,11,23} + \overline{7,13,23} + \overline{7,11,25} + \overline{5,9,29} + \overline{5,7,15,17} + \overline{3,9,15,17} + \overline{5,7,13,19} + \overline{3,9,13,19} + \overline{3,5,17,19} + \\
& \overline{5,7,11,21} + \overline{3,9,11,21} + \overline{3,5,15,21} + \overline{5,7,9,23} + \overline{3,5,13,23} + \overline{3,7,9,25} + \overline{3,5,11,25}|_{44} + \overline{9,17,19} + \overline{9,15,21} + \\
& \overline{5,19,21} + \overline{9,13,23} + \overline{5,17,23} + \overline{9,11,25} + \overline{5,15,25} + \overline{5,13,27} + \overline{7,9,29} + \overline{5,11,29} + \overline{5,9,31} + \overline{5,9,15,17} + \overline{5,9,13,19} + \\
& \overline{5,9,11,21} + \overline{5,7,9,25} + \overline{3,5,9,29}|_{46} + \overline{5,7,9,11,15} + \overline{3,7,9,13,15} + \overline{3,5,11,13,15} + \overline{3,5,7,15,17} + \overline{3,5,7,13,19} + \\
& \overline{13,15,19} + \overline{3,5,7,11,21} + \overline{7,19,21} + \overline{3,5,7,9,23} + \overline{9,15,23} + \overline{7,17,23} + \overline{7,15,25} + \overline{7,13,27} + \overline{5,15,27} + \overline{7,11,29} + \\
& \overline{7,9,31} + \overline{5,11,31} + \overline{5,9,33} + \overline{3,5,7,9,11,13} + \overline{7,9,15,17} + \overline{3,13,15,17} + \overline{5,11,13,19} + \overline{7,9,11,21} + \overline{3,11,13,21} + \\
& \overline{3,5,19,21} + \overline{5,9,11,23} + \overline{3,5,17,23} + \overline{3,7,13,25} + \overline{3,5,15,25} + \overline{3,5,13,27} + \overline{3,7,9,29} + \overline{3,5,11,29} + \overline{3,5,9,31}|_{48} + \\
& \overline{5,7,9,11,17} + \overline{3,7,9,13,17} + \overline{3,5,11,13,17} + \overline{3,5,9,15,17} + \overline{3,5,9,13,19} + \overline{13,17,19} + \overline{3,5,9,11,21} + \overline{13,15,21} + \\
& \overline{9,19,21} + \overline{11,15,23} + \overline{3,5,7,9,25} + \overline{9,13,27} + \overline{7,15,27} + \overline{5,17,27} + \overline{9,11,29} + \overline{5,15,29} + \overline{7,11,31} + \overline{5,13,31} + \overline{7,9,33} + \\
& \overline{5,11,33} + \overline{7,11,15,17} + \overline{5,13,15,17} + \overline{7,11,13,19} + \overline{7,9,15,19} + \overline{5,11,15,19} + \overline{7,9,13,21} + \overline{3,11,15,21} + \overline{5,9,13,23} + \\
& \overline{3,11,13,23} + \overline{3,9,15,23} + \overline{5,9,11,25} + \overline{5,7,13,25} + \overline{3,9,13,25} + \overline{3,7,15,25} + \overline{3,7,13,27} + \overline{3,5,15,27} + \overline{5,7,9,29} + \\
& \overline{3,7,11,29} + \overline{3,7,9,31} + \overline{3,5,11,31} + \overline{3,5,9,33}|_{50} + \overline{13,17,21} + \overline{13,15,23} + \overline{11,17,23} + \overline{11,15,25} + \overline{9,17,25} + \overline{9,15,27} + \\
& \overline{7,17,27} + \overline{7,15,29} + \overline{5,17,29} + \overline{9,11,31} + \overline{7,13,31} + \overline{7,11,33} + \overline{5,13,33} + \overline{9,11,15,17} + \overline{7,13,15,17} + \overline{9,11,13,19} + \\
& \overline{5,13,15,19} + \overline{7,9,17,19} + \overline{5,11,17,19} + \overline{7,11,13,21} + \overline{7,9,15,21} + \overline{3,13,15,21} + \overline{3,11,17,21} + \overline{5,11,13,23} + \overline{5,9,15,23} + \\
& \overline{3,9,17,23} + \overline{3,11,13,25} + \overline{5,7,15,25} + \overline{3,7,17,25} + \overline{5,7,13,27} + \overline{3,9,13,27} + \overline{3,5,17,27} + \overline{5,7,11,29} + \overline{3,9,11,29} + \\
& \overline{3,5,15,29} + \overline{5,7,9,31} + \overline{3,5,13,31} + \overline{3,7,9,33} + \overline{3,5,11,33}|_{52} + \overline{5,7,9,15,17} + \overline{3,5,13,15,17} + \overline{5,7,9,13,19} + \\
& \overline{5,7,9,11,21} + \overline{3,5,11,13,21} + \overline{15,17,21} + \overline{3,5,9,13,23} + \overline{13,17,23} + \overline{3,5,7,13,25} + \overline{11,17,25} + \overline{5,23,25} + \overline{9,17,27} + \\
& \overline{5,21,27} + \overline{3,5,7,9,29} + \overline{7,17,29} + \overline{5,19,29} + \overline{5,17,31} + \overline{9,11,33} + \overline{7,13,33} + \overline{5,13,17,19} + \overline{5,11,17,21} + \overline{5,9,17,23} + \\
& \overline{5,7,17,25} + \overline{3,5,17,29} + \overline{5,7,9,33} + \overline{3,5,13,33}
\end{aligned}
$$

$$
\begin{aligned}
e_2 \quad = \quad & \overline{5}|_6 + \overline{7} + \overline{3,5}|_8 + \overline{9} + \overline{3,7}|_{10} + \overline{5,7} + \overline{3,9}|_{12} + \overline{5,9}|_{14} + \overline{3,5,7} + \overline{15} + \overline{5,11}|_{16} + \overline{3,5,9} + \overline{17} + \overline{7,11} + \overline{5,13} + \overline{3,15}|_{18} + \overline{9,11} + \overline{7,13} + \\
& \overline{5,15} + \overline{3,17}|_{20} + \overline{5,7,9} + \overline{3,5,13} + \overline{5,17}|_{22} + \overline{5,7,11} + \overline{3,7,13} + \overline{3,5,15} + \overline{3,5,7,9} + \overline{11,13} + \overline{9,15}|_{24} + \overline{5,9,11} + \overline{3,9,13} + \\
& \overline{3,5,17} + \overline{9,17}|_{26} + \overline{5,9,13}|_{28} + \overline{7,9,13} + \overline{5,11,13} + \overline{5,9,15} + \overline{3,5,9,13}|_{30} + \overline{7,11,13} + \overline{3,13,15} + \overline{5,9,17} + \overline{31} + \overline{5,7,9,11} + \\
& \overline{3,5,9,15} + \overline{13,19} + \overline{9,23} + \overline{5,27}|_{32} + \overline{9,11,13} + \overline{7,11,15} + \overline{5,13,15} + \overline{3,13,17} + \overline{33} + \overline{5,7,9,13} + \overline{3,7,11,13} + \overline{3,7,9,15} + \\
& \overline{3,5,11,15} + \overline{3,5,9,17} + \overline{15,19} + \overline{13,21} + \overline{11,23} + \overline{9,25} + \overline{7,27} + \overline{5,29} + \overline{3,31}|_{34} + \overline{9,11,15} + \overline{7,13,15} + \overline{7,11,17} + \overline{5,13,17} + \\
& \overline{5,7,11,13} + \overline{3,9,11,13} + \overline{5,7,9,15} + \overline{3,5,13,15} + \overline{3,7,9,17} + \overline{3,5,11,17} + \overline{17,19} + \overline{15,21} + \overline{13,23} + \overline{11,25} + \overline{9,27} + \overline{7,29} + \\
& \overline{5,31} + \overline{3,33}|_{36} + \overline{3,5,7,9,13} + \overline{9,11,17} + \overline{7,13,17} + \overline{5,15,17} + \overline{5,11,21} + \overline{5,7,25} + \overline{3,5,29} + \overline{5,7,9,17} + \overline{3,5,13,17} + \\
& \overline{5,33}|_{38} + \overline{3,5,7,11,13} + \overline{3,5,7,9,15} + \overline{11,13,15} + \overline{7,15,17} + \overline{5,15,19} + \overline{7,11,21} + \overline{5,13,21} + \overline{5,11,23} + \overline{5,9,25} + \\
& \overline{5,7,27} + \overline{3,7,29} + \overline{3,5,31} + \overline{7,9,11,13} + \overline{5,9,11,15} + \overline{3,9,13,15} + \overline{3,5,15,17} + \overline{3,5,11,21} + \overline{19,21} + \overline{17,23} + \overline{3,5,7,25} + \\
& \overline{15,25} + \overline{13,27} + \overline{11,29} + \overline{9,31}|_{40} + \overline{3,5,9,11,13} + \overline{3,5,7,9,17} + \overline{11,13,17} + \overline{9,15,17} + \overline{7,15,19} + \overline{5,17,19} + \overline{9,11,21} + \\
& \overline{7,13,21} + \overline{5,15,21} + \overline{7,11,23} + \overline{5,13,23} + \overline{5,11,25} + \overline{5,9,27} + \overline{3,9,29} + \overline{3,5,33} + \overline{5,9,11,17} + \overline{3,9,13,17} + \overline{3,7,15,17} + \\
& \overline{3,7,13,19} + \overline{3,5,15,19} + \overline{3,7,11,21} + \overline{3,5,13,21} + \overline{3,7,9,23} + \overline{3,5,11,23} + \overline{3,5,9,25} + \overline{9,33}|_{42} + \overline{9,15,19} + \overline{7,17,19} + \\
& \overline{9,13,21} + \overline{7,15,21} + \overline{9,11,23} + \overline{7,13,23} + \overline{7,11,25} + \overline{5,9,29} + \overline{5,7,15,17} + \overline{3,9,15,17} + \overline{5,7,13,19} + \overline{3,9,13,19} + \\
& \overline{3,5,17,19} + \overline{5,7,11,21} + \overline{3,9,11,21} + \overline{3,5,15,21} + \overline{5,7,9,23} + \overline{3,5,13,23} + \overline{3,7,9,25} + \overline{3,5,11,25}|_{44} + \overline{9,17,19} + \\
& \overline{9,15,21} + \overline{5,19,21} + \overline{9,13,23} + \overline{5,17,23} + \overline{9,11,25} + \overline{5,15,25} + \overline{5,13,27} + \overline{7,9,29} + \overline{5,11,29} + \overline{5,9,31} + \overline{5,9,15,17} + \\
& \overline{5,9,13,19} + \overline{5,9,11,21} + \overline{5,7,9,25} + \overline{3,5,9,29}|_{46} + \overline{5,7,9,11,15} + \overline{3,7,9,13,15} + \overline{3,5,11,13,15} + \overline{3,5,7,15,17} + \\
& \overline{3,5,7,13,19} + \overline{3,5,7,11,21} + \overline{11,15,21} + \overline{7,19,21} + \overline{3,5,7,9,23} + \overline{7,17,23} + \overline{7,13,27} + \overline{7,11,29} + \overline{3,15,29} + \overline{3,13,31} + \\
& \overline{5,9,33} + \overline{3,5,7,9,11,13} + \overline{5,11,15,17} + \overline{7,9,13,19} + \overline{3,5,19,21} + \overline{3,9,13,23} + \overline{3,5,17,23} + \overline{5,7,11,25} + \overline{3,5,15,25} + \\
& \overline{23,25} + \overline{5,7,9,27} + \overline{21,27} + \overline{19,29} + \overline{3,5,9,31} + \overline{17,31}|_{48} + \overline{5,7,9,11,17} + \overline{3,7,9,13,17} + \overline{3,5,11,13,17} + \overline{3,5,9,15,17} + \\
& \overline{3,5,9,13,19} + \overline{3,5,9,11,21} + \overline{13,15,21} + \overline{11,17,21} + \overline{9,19,21} + \overline{11,15,23} + \overline{9,17,23} + \overline{3,5,7,9,25} + \overline{7,17,25} + \overline{9,13,27} + \\
& \overline{7,15,27} + \overline{9,11,29} + \overline{5,15,29} + \overline{3,17,29} + \overline{7,11,31} + \overline{5,13,31} + \overline{3,13,33} + \overline{7,11,15,17} + \overline{5,13,15,17} + \overline{7,11,13,19} + \\
& \overline{7,9,15,19} + \overline{5,11,15,19} + \overline{7,9,13,21} + \overline{3,11,15,21} + \overline{5,9,13,23} + \overline{3,11,13,23} + \overline{3,9,15,23} + \overline{5,9,11,25} + \overline{5,7,13,25} + \\
& \overline{3,9,13,25} + \overline{3,7,15,25} + \overline{3,7,13,27} + \overline{3,5,15,27} + \overline{5,7,9,29} + \overline{3,7,11,29} + \overline{3,7,9,31} + \overline{3,5,11,31} + \overline{3,5,9,33} + \overline{17,33}|_{50} + \\
& \overline{13,17,21} + \overline{13,15,23} + \overline{11,17,23} + \overline{11,15,25} + \overline{9,17,25} + \overline{9,15,27} + \overline{7,17,27} + \overline{7,15,29} + \overline{5,17,29} + \overline{9,11,31} + \overline{7,13,31} + \\
& \overline{7,11,33} + \overline{5,13,33} + \overline{9,11,15,17} + \overline{7,13,15,17} + \overline{9,11,13,19} + \overline{5,13,15,19} + \overline{7,9,17,19} + \overline{5,11,17,19} + \overline{7,11,13,21} + \\
& \overline{7,9,15,21} + \overline{3,13,15,21} + \overline{3,11,17,21} + \overline{5,11,13,23} + \overline{5,9,15,23} + \overline{3,9,17,23} + \overline{3,11,13,25} + \overline{5,7,15,25} + \overline{3,7,17,25} + \\
& \overline{5,7,13,27} + \overline{3,9,13,27} + \overline{3,5,17,27} + \overline{5,7,11,29} + \overline{3,9,11,29} + \overline{3,5,15,29} + \overline{5,7,9,31} + \overline{3,5,13,31} + \overline{3,7,9,33} + \\
& \overline{3,5,11,33}|_{52} + \overline{5,7,9,15,17} + \overline{3,5,13,15,17} + \overline{5,7,9,13,19} + \overline{5,7,9,11,21} + \overline{3,5,11,13,21} + \overline{15,17,21} + \overline{3,5,9,13,23} + \\
& \overline{13,17,23} + \overline{3,5,7,13,25} + \overline{11,17,25} + \overline{5,23,25} + \overline{9,17,27} + \overline{5,21,27} + \overline{3,5,7,9,29} + \overline{7,17,29} + \overline{5,19,29} + \overline{5,17,31} + \\
& \overline{9,11,33} + \overline{7,13,33} + \overline{5,13,17,19} + \overline{5,11,17,21} + \overline{5,9,17,23} + \overline{5,7,17,25} + \overline{3,5,17,29} + \overline{5,7,9,33} + \overline{3,5,13,33}
\end{aligned}
$$

$$
\begin{aligned}
e_3 \quad = \quad & \overline{3,7}|_{10} + \overline{5,7} + \overline{3,9}|_{12} + \overline{5,9}|_{14} + \overline{3,5,7} + \overline{7,9} + \overline{3,13}|_{16} + \overline{3,5,9} + \overline{7,11} + \overline{5,13} + \overline{3,15}|_{18} + \overline{9,11} + \overline{7,13} + \overline{5,15} + \overline{3,17}|_{20} + \\
& \overline{5,7,9} + \overline{3,5,13} + \overline{5,17}|_{22} + \overline{5,7,11} + \overline{3,7,13} + \overline{3,5,15} + \overline{3,5,7,9} + \overline{11,13} + \overline{9,15}|_{24} + \overline{5,9,11} + \overline{3,9,13} + \overline{3,5,17} + \overline{9,17}|_{26} + \\
& \overline{5,9,13}|_{28} + \overline{7,9,13} + \overline{5,11,13} + \overline{5,9,15} + \overline{3,5,9,13}|_{30} + \overline{7,11,13} + \overline{3,13,15} + \overline{5,9,17} + \overline{5,7,9,11} + \overline{3,5,9,15} + \overline{15,17} + \\
& \overline{11,21} + \overline{7,25} + \overline{3,29}|_{32} + \overline{9,11,13} + \overline{7,11,15} + \overline{5,13,15} + \overline{3,13,17} + \overline{5,7,9,13} + \overline{3,7,11,13} + \overline{3,7,9,15} + \overline{3,5,11,15} + \\
& \overline{3,5,9,17} + \overline{15,19} + \overline{13,21} + \overline{11,23} + \overline{9,25} + \overline{7,27} + \overline{5,29} + \overline{3,31}|_{34} + \overline{9,11,15} + \overline{7,13,15} + \overline{7,11,17} + \overline{5,13,17} + \overline{5,7,11,13} + \\
& \overline{3,9,11,13} + \overline{5,7,9,15} + \overline{3,5,13,15} + \overline{3,7,9,17} + \overline{3,5,11,17} + \overline{17,19} + \overline{15,21} + \overline{13,23} + \overline{11,25} + \overline{9,27} + \overline{7,29} + \overline{5,31} + \\
& \overline{3,33}|_{36} + \overline{3,5,7,9,13} + \overline{9,11,17} + \overline{7,13,17} + \overline{5,15,17} + \overline{5,11,21} + \overline{5,7,25} + \overline{3,5,29} + \overline{5,7,9,17} + \overline{3,5,13,17} + \overline{5,33}|_{38} + \\
& \overline{3,5,7,11,13} + \overline{3,5,7,9,15} + \overline{11,13,15} + \overline{7,15,17} + \overline{5,15,19} + \overline{7,11,21} + \overline{5,13,21} + \overline{5,11,23} + \overline{5,9,25} + \overline{5,7,27} + \\
& \overline{3,7,29} + \overline{3,5,31} + \overline{7,9,11,13} + \overline{5,9,11,15} + \overline{3,9,13,15} + \overline{3,5,15,17} + \overline{3,5,11,21} + \overline{19,21} + \overline{17,23} + \overline{3,5,7,25} + \overline{15,25} + \\
& \overline{13,27} + \overline{11,29} + \overline{9,31}|_{40} + \overline{3,5,9,11,13} + \overline{3,5,7,9,17} + \overline{11,13,17} + \overline{9,15,17} + \overline{7,15,19} + \overline{5,17,19} + \overline{9,11,21} + \overline{7,13,21} + \\
& \overline{5,15,21} + \overline{7,11,23} + \overline{5,13,23} + \overline{5,11,25} + \overline{5,9,27} + \overline{3,9,29} + \overline{3,5,33} + \overline{5,9,11,17} + \overline{3,9,13,17} + \overline{3,7,15,17} + \overline{3,7,13,19} + \\
& \overline{3,5,15,19} + \overline{3,7,11,21} + \overline{3,5,13,21} + \overline{3,7,9,23} + \overline{3,5,11,23} + \overline{3,5,9,25} + \overline{9,33}|_{42} + \overline{9,15,19} + \overline{7,17,19} + \overline{9,13,21} + \\
& \overline{7,15,21} + \overline{9,11,23} + \overline{7,13,23} + \overline{7,11,25} + \overline{5,9,29} + \overline{5,7,15,17} + \overline{3,9,15,17} + \overline{5,7,13,19} + \overline{3,9,13,19} + \overline{3,5,17,19} + \\
& \overline{5,7,11,21} + \overline{3,9,11,21} + \overline{3,5,15,21} + \overline{5,7,9,23} + \overline{3,5,13,23} + \overline{3,7,9,25} + \overline{3,5,11,25}|_{44} + \overline{9,17,19} + \overline{9,15,21} + \overline{5,19,21} + \\
& \overline{9,13,23} + \overline{5,17,23} + \overline{9,11,25} + \overline{5,15,25} + \overline{5,13,27} + \overline{7,9,29} + \overline{5,11,29} + \overline{5,9,31} + \overline{5,9,15,17} + \overline{5,9,13,19} + \overline{5,9,11,21} + \\
& \overline{5,7,9,25} + \overline{3,5,9,29}|_{46} + \overline{5,7,9,11,15} + \overline{3,7,9,13,15} + \overline{3,5,11,13,15} + \overline{3,5,7,15,17} + \overline{3,5,7,13,19} + \overline{3,5,7,11,21} + \\
& \overline{11,15,21} + \overline{7,19,21} + \overline{3,5,7,9,23} + \overline{7,17,23} + \overline{7,13,27} + \overline{7,11,29} + \overline{3,15,29} + \overline{3,13,31} + \overline{5,9,33} + \overline{3,5,7,9,11,13} + \\
& \overline{5,11,15,17} + \overline{7,9,13,19} + \overline{3,5,19,21} + \overline{3,9,13,23} + \overline{3,5,17,23} + \overline{5,7,11,25} + \overline{3,5,15,25} + \overline{23,25} + \overline{5,7,9,27} + \\
& \overline{21,27} + \overline{19,29} + \overline{3,5,9,31} + \overline{17,31}|_{48} + \overline{5,7,9,11,17} + \overline{3,7,9,13,17} + \overline{3,5,11,13,17} + \overline{3,5,9,15,17} + \overline{3,5,9,13,19} + \\
& \overline{3,5,9,11,21} + \overline{13,15,21} + \overline{11,17,21} + \overline{9,19,21} + \overline{11,15,23} + \overline{9,17,23} + \overline{3,5,7,9,25} + \overline{7,17,25} + \overline{9,13,27} + \overline{7,15,27} + \\
& \overline{9,11,29} + \overline{5,15,29} + \overline{3,17,29} + \overline{7,11,31} + \overline{5,13,31} + \overline{3,13,33} + \overline{7,11,15,17} + \overline{5,13,15,17} + \overline{7,11,13,19} + \overline{7,9,15,19} + \\
& \overline{5,11,15,19} + \overline{7,9,13,21} + \overline{3,11,15,21} + \overline{5,9,13,23} + \overline{3,11,13,23} + \overline{3,9,15,23} + \overline{5,9,11,25} + \overline{5,7,13,25} + \overline{3,9,13,25} + \\
& \overline{3,7,15,25} + \overline{3,7,13,27} + \overline{3,5,15,27} + \overline{5,7,9,29} + \overline{3,7,11,29} + \overline{3,7,9,31} + \overline{3,5,11,31} + \overline{3,5,9,33} + \overline{17,33}|_{50} + \overline{13,17,21} + \\
& \overline{13,15,23} + \overline{11,17,23} + \overline{11,15,25} + \overline{9,17,25} + \overline{9,15,27} + \overline{7,17,27} + \overline{7,15,29} + \overline{5,17,29} + \overline{9,11,31} + \overline{7,13,31} + \overline{7,11,33} + \\
& \overline{5,13,33} + \overline{9,11,15,17} + \overline{7,13,15,17} + \overline{9,11,13,19} + \overline{5,13,15,19} + \overline{7,9,17,19} + \overline{5,11,17,19} + \overline{7,11,13,21} + \overline{7,9,15,21} + \\
& \overline{3,13,15,21} + \overline{3,11,17,21} + \overline{5,11,13,23} + \overline{5,9,15,23} + \overline{3,9,17,23} + \overline{3,11,13,25} + \overline{5,7,15,25} + \overline{3,7,17,25} + \overline{5,7,13,27} + \\
& \overline{3,9,13,27} + \overline{3,5,17,27} + \overline{5,7,11,29} + \overline{3,9,11,29} + \overline{3,5,15,29} + \overline{5,7,9,31} + \overline{3,5,13,31} + \overline{3,7,9,33} + \overline{3,5,11,33}|_{52} + \\
& \overline{5,7,9,15,17} + \overline{3,5,13,15,17} + \overline{5,7,9,13,19} + \overline{5,7,9,11,21} + \overline{3,5,11,13,21} + \overline{15,17,21} + \overline{3,5,9,13,23} + \overline{13,17,23} + \\
& \overline{3,5,7,13,25} + \overline{11,17,25} + \overline{5,23,25} + \overline{9,17,27} + \overline{5,21,27} + \overline{3,5,7,9,29} + \overline{7,17,29} + \overline{5,19,29} + \overline{5,17,31} + \overline{9,11,33} + \\
& \overline{7,13,33} + \overline{5,13,17,19} + \overline{5,11,17,21} + \overline{5,9,17,23} + \overline{5,7,17,25} + \overline{3,5,17,29} + \overline{5,7,9,33} + \overline{3,5,13,33}
\end{aligned}
$$

$$
\begin{aligned}
e_4 \quad = \quad & \overline{5,9,13}|_{28} + \overline{7,9,13} + \overline{5,11,13} + \overline{5,9,15} + \overline{3,5,9,13}|_{30} + \overline{7,11,13} + \overline{7,9,15} + \overline{5,11,15} + \overline{5,9,17} + \overline{3,7,9,13} + \overline{3,5,11,13} + \\
& \overline{3,5,9,15}|_{32} + \overline{9,11,13} + \overline{7,11,15} + \overline{5,13,15} + \overline{7,9,17} + \overline{5,11,17} + \overline{5,7,9,13} + \overline{3,7,11,13} + \overline{3,7,9,15} + \overline{3,5,11,15} + \\
& \overline{3,5,9,17}|_{34} + \overline{9,11,15} + \overline{7,13,15} + \overline{7,11,17} + \overline{5,13,17} + \overline{5,7,11,13} + \overline{3,9,11,13} + \overline{5,7,9,15} + \overline{3,7,11,15} + \overline{3,5,13,15} + \\
& \overline{3,7,9,17} + \overline{3,5,11,17}|_{36} + \overline{3,5,7,9,13} + \overline{9,11,17} + \overline{7,13,17} + \overline{5,13,19} + \overline{5,9,23} + \overline{5,7,11,15} + \overline{3,9,11,15} + \overline{3,7,13,15} + \\
& \overline{5,7,9,17} + \overline{3,7,11,17} + \overline{3,5,13,17}|_{38} + \overline{3,5,7,11,13} + \overline{3,5,7,9,15} + \overline{11,13,15} + \overline{7,13,19} + \overline{5,15,19} + \overline{5,13,21} + \\
& \overline{7,9,23} + \overline{5,11,23} + \overline{5,9,25} + \overline{7,9,11,13} + \overline{5,7,13,15} + \overline{5,7,11,17} + \overline{3,9,11,17} + \overline{3,7,13,17} + \overline{3,5,13,19} + \overline{3,5,9,23}|_{40} + \\
& \overline{3,5,9,11,13} + \overline{3,5,7,11,15} + \overline{3,5,7,9,17} + \overline{11,13,17} + \overline{9,13,19} + \overline{7,15,19} + \overline{5,17,19} + \overline{7,13,21} + \overline{5,15,21} + \overline{7,11,23} + \\
& \overline{5,13,23} + \overline{7,9,25} + \overline{5,11,25} + \overline{7,9,11,15} + \overline{5,9,13,15} + \overline{3,11,13,15} + \overline{5,7,13,17} + \overline{3,7,13,19} + \overline{3,5,15,19} + \overline{3,5,13,21} + \\
& \overline{3,7,9,23} + \overline{3,5,11,23} + \overline{3,5,9,25}|_{42} + \overline{3,5,9,11,15} + \overline{3,5,7,13,15} + \overline{3,5,7,11,17} + \overline{9,15,19} + \overline{7,17,19} + \overline{9,13,21} + \\
& \overline{7,15,21} + \overline{9,11,23} + \overline{7,13,23} + \overline{7,11,25} + \overline{5,9,29} + \overline{7,9,13,15} + \overline{5,11,13,15} + \overline{7,9,11,17} + \overline{5,9,13,17} + \overline{3,11,13,17} + \\
& \overline{5,7,13,19} + \overline{3,9,13,19} + \overline{3,7,15,19} + \overline{3,5,17,19} + \overline{3,7,13,21} + \overline{3,5,15,21} + \overline{5,7,9,23} + \overline{3,7,11,23} + \overline{3,5,13,23} + \\
& \overline{3,7,9,25} + \overline{3,5,11,25}|_{44} + \overline{3,5,9,13,15} + \overline{3,5,9,11,17} + \overline{3,5,7,13,17} + \overline{9,17,19} + \overline{9,15,21} + \overline{5,19,21} + \overline{9,13,23} + \\
& \overline{5,17,23} + \overline{9,11,25} + \overline{5,15,25} + \overline{5,13,27} + \overline{7,9,29} + \overline{5,11,29} + \overline{5,9,31} + \overline{7,9,13,17} + \overline{5,11,13,17} + \overline{5,9,13,19} + \overline{5,7,15,19} + \\
& \overline{3,9,15,19} + \overline{3,7,17,19} + \overline{5,7,13,21} + \overline{3,9,13,21} + \overline{3,7,15,21} + \overline{5,7,11,23} + \overline{3,9,11,23} + \overline{3,7,13,23} + \overline{5,7,9,25} + \\
& \overline{3,7,11,25} + \overline{3,5,9,29}|_{46} + \overline{3,7,9,13,15} + \overline{3,5,9,13,17} + \overline{3,5,7,13,19} + \overline{13,15,19} + \overline{7,19,21} + \overline{3,5,7,9,23} + \overline{9,15,23} + \\
& \overline{7,17,23} + \overline{7,15,25} + \overline{7,13,27} + \overline{5,15,27} + \overline{7,11,29} + \overline{7,9,31} + \overline{5,11,31} + \overline{5,9,33} + \overline{3,5,7,9,11,13} + \overline{9,11,13,15} + \\
& \overline{5,11,13,19} + \overline{5,9,15,19} + \overline{5,7,17,19} + \overline{3,9,17,19} + \overline{5,9,13,21} + \overline{5,7,15,21} + \overline{3,9,15,21} + \overline{3,5,19,21} + \overline{5,7,13,23} + \\
& \overline{3,9,13,23} + \overline{3,5,17,23} + \overline{5,7,11,25} + \overline{3,9,11,25} + \overline{3,5,15,25} + \overline{3,5,13,27} + \overline{3,7,9,29} + \overline{3,5,11,29} + \overline{3,5,9,31}|_{48} + \\
& \overline{5,7,9,13,15} + \overline{3,7,11,13,15} + \overline{3,7,9,13,17} + \overline{3,5,9,13,19} + \overline{3,5,7,15,19} + \overline{13,17,19} + \overline{3,5,7,13,21} + \overline{13,15,21} + \\
& \overline{9,19,21} + \overline{3,5,7,11,23} + \overline{11,15,23} + \overline{3,5,7,9,25} + \overline{9,13,27} + \overline{7,15,27} + \overline{5,17,27} + \overline{9,11,29} + \overline{5,15,29} + \overline{7,11,31} + \\
& \overline{5,13,31} + \overline{7,9,33} + \overline{5,11,33} + \overline{3,5,7,9,11,15} + \overline{9,11,13,17} + \overline{7,11,13,19} + \overline{5,11,15,19} + \overline{3,13,15,19} + \overline{5,9,17,19} + \\
& \overline{5,11,13,21} + \overline{5,9,15,21} + \overline{3,7,19,21} + \overline{7,9,11,23} + \overline{3,9,15,23} + \overline{3,7,17,23} + \overline{3,7,15,25} + \overline{3,7,13,27} + \overline{3,5,15,27} + \\
& \overline{5,7,9,29} + \overline{3,7,11,29} + \overline{3,7,9,31} + \overline{3,5,11,31} + \overline{3,5,9,33}|_{50} + \overline{5,7,11,13,15} + \overline{3,9,11,13,15} + \overline{5,7,9,13,17} + \\
& \overline{3,7,11,13,17} + \overline{3,5,9,15,19} + \overline{3,5,7,17,19} + \overline{3,5,9,13,21} + \overline{3,5,7,15,21} + \overline{13,17,21} + \overline{3,5,9,11,23} + \overline{3,5,7,13,23} + \\
& \overline{13,15,23} + \overline{11,17,23} + \overline{3,5,7,11,25} + \overline{11,15,25} + \overline{9,17,25} + \overline{9,15,27} + \overline{7,17,27} + \overline{7,15,29} + \overline{5,17,29} + \overline{9,11,31} + \\
& \overline{7,13,31} + \overline{7,11,33} + \overline{5,13,33} + \overline{3,5,7,9,13,15} + \overline{3,5,7,9,11,17} + \overline{9,11,13,19} + \overline{7,11,15,19} + \overline{5,11,17,19} + \overline{3,13,17,19} + \\
& \overline{7,11,13,21} + \overline{5,11,15,21} + \overline{3,13,15,21} + \overline{5,7,19,21} + \overline{3,9,19,21} + \overline{7,9,13,23} + \overline{5,9,15,23} + \overline{3,11,15,23} + \overline{5,7,17,23} + \\
& \overline{7,9,11,25} + \overline{5,9,13,25} + \overline{5,7,15,25} + \overline{5,7,13,27} + \overline{3,9,13,27} + \overline{3,7,15,27} + \overline{3,5,17,27} + \overline{5,7,11,29} + \overline{3,9,11,29} + \\
& \overline{3,5,15,29} + \overline{5,7,9,31} + \overline{3,7,11,31} + \overline{3,5,13,31} + \overline{3,7,9,33} + \overline{3,5,11,33}|_{52} + \overline{5,7,11,13,17} + \overline{3,9,11,13,17} + \\
& \overline{5,7,9,13,19} + \overline{3,5,9,17,19} + \overline{3,5,9,15,21} + \overline{15,17,21} + \overline{13,17,23} + \overline{3,5,9,11,25} + \overline{11,17,25} + \overline{5,23,25} + \overline{9,17,27} + \\
& \overline{5,21,27} + \overline{3,5,7,9,29} + \overline{7,17,29} + \overline{5,19,29} + \overline{5,17,31} + \overline{9,11,33} + \overline{7,13,33} + \overline{3,5,7,9,13,17} + \overline{9,11,15,19} + \overline{7,13,15,19} + \\
& \overline{7,11,17,19} + \overline{9,11,13,21} + \overline{7,11,15,21} + \overline{3,13,17,21} + \overline{5,9,19,21} + \overline{5,11,15,23} + \overline{3,13,15,23} + \overline{3,11,17,23} + \\
& \overline{7,9,13,25} + \overline{5,11,13,25} + \overline{3,11,15,25} + \overline{3,9,17,25} + \overline{5,7,15,27} + \overline{3,9,15,27} + \overline{3,7,17,27} + \overline{3,7,15,29} + \overline{3,5,17,29} + \\
& \overline{5,7,11,31} + \overline{3,9,11,31} + \overline{3,7,13,31} + \overline{5,7,9,33} + \overline{3,7,11,33} + \overline{3,5,13,33}
\end{aligned}
$$

$$
\begin{aligned}
e_5 \quad = \quad & \overline{3,7,11,15}|_{36}+\overline{5,7,11,15}+\overline{3,9,11,15}+\overline{3,7,13,15}+\overline{3,7,11,17}|_{38}+\overline{5,9,11,15}+\overline{5,7,13,15}+\overline{3,9,13,15}+\overline{5,7,11,17}+\\
& \overline{3,9,11,17}+\overline{3,7,13,17}|_{40}+\overline{3,5,7,11,15}+\overline{7,9,11,15}+\overline{5,9,13,15}+\overline{3,11,13,15}+\overline{5,9,11,17}+\overline{5,7,13,17}+\overline{3,9,13,17}+\\
& \overline{3,7,15,17}+\overline{3,7,11,21}|_{42}+\overline{3,5,9,11,15}+\overline{3,5,7,13,15}+\overline{3,5,7,11,17}+\overline{7,9,13,15}+\overline{5,11,13,15}+\overline{7,9,11,17}+\\
& \overline{5,9,13,17}+\overline{3,11,13,17}+\overline{5,7,15,17}+\overline{3,9,15,17}+\overline{3,7,15,19}+\overline{5,7,11,21}+\overline{3,9,11,21}+\overline{3,7,13,21}+\overline{3,7,11,23}|_{44}+\\
& \overline{3,5,9,13,15}+\overline{3,5,9,11,17}+\overline{3,5,7,13,17}+\overline{7,9,13,17}+\overline{5,11,13,17}+\overline{5,9,15,17}+\overline{5,7,15,19}+\overline{3,9,15,19}+\\
& \overline{3,7,17,19}+\overline{5,9,11,21}+\overline{5,7,13,21}+\overline{3,9,13,21}+\overline{3,7,15,21}+\overline{5,7,11,23}+\overline{3,9,11,23}+\overline{3,7,13,23}+\overline{3,7,11,25}|_{46}+\\
& \overline{5,7,9,11,15}+\overline{3,5,11,13,15}+\overline{3,5,9,13,17}+\overline{3,5,7,15,17}+\overline{3,5,7,11,21}+\overline{9,11,13,15}+\overline{7,9,15,17}+\overline{3,13,15,17}+\\
& \overline{5,9,15,19}+\overline{5,7,17,19}+\overline{3,9,17,19}+\overline{7,9,11,21}+\overline{5,9,13,21}+\overline{3,11,13,21}+\overline{5,7,15,21}+\overline{3,9,15,21}+\overline{5,9,11,23}+\\
& \overline{5,7,13,23}+\overline{3,9,13,23}+\overline{5,7,11,25}+\overline{3,9,11,25}+\overline{3,7,13,25}|_{48}+\overline{5,7,9,13,15}+\overline{3,7,11,13,15}+\overline{5,7,9,11,17}+\\
& \overline{3,5,11,13,17}+\overline{3,5,9,15,17}+\overline{3,5,7,15,19}+\overline{3,5,9,11,21}+\overline{3,5,7,13,21}+\overline{3,5,7,11,23}+\overline{3,5,7,9,11,15}+\\
& \overline{9,11,13,17}+\overline{7,11,15,17}+\overline{5,13,15,17}+\overline{7,9,15,19}+\overline{3,13,15,19}+\overline{5,9,17,19}+\overline{7,9,13,21}+\overline{5,11,13,21}+\overline{5,9,15,21}+\\
& \overline{3,11,15,21}+\overline{3,7,19,21}+\overline{7,9,11,23}+\overline{5,9,13,23}+\overline{3,11,13,23}+\overline{3,7,17,23}+\overline{5,9,11,25}+\overline{5,7,13,25}+\overline{3,9,13,25}|_{50}+\\
& \overline{5,7,11,13,15}+\overline{3,9,11,13,15}+\overline{5,7,9,13,17}+\overline{3,7,11,13,17}+\overline{3,5,9,15,19}+\overline{3,5,7,17,19}+\overline{3,5,9,13,21}+\\
& \overline{3,5,7,15,21}+\overline{3,5,9,11,23}+\overline{3,5,7,13,23}+\overline{3,5,7,11,25}+\overline{3,5,7,9,13,15}+\overline{3,5,7,9,11,17}+\overline{9,11,15,17}+\\
& \overline{7,13,15,17}+\overline{7,11,15,19}+\overline{5,13,15,19}+\overline{7,9,17,19}+\overline{3,13,17,19}+\overline{7,9,15,21}+\overline{5,11,15,21}+\overline{3,11,17,21}+\overline{5,7,19,21}+\\
& \overline{3,9,19,21}+\overline{7,9,13,23}+\overline{5,11,13,23}+\overline{3,11,15,23}+\overline{5,7,17,23}+\overline{3,9,17,23}+\overline{7,9,11,25}+\overline{5,9,13,25}+\overline{3,11,13,25}+\\
& \overline{3,7,17,25}+\overline{3,7,15,27}+\overline{3,7,11,31}|_{52}+\overline{5,7,11,13,17}+\overline{3,9,11,13,17}+\overline{5,7,9,15,17}+\overline{3,5,13,15,17}+\overline{3,5,9,17,19}+\\
& \overline{5,7,9,11,21}+\overline{3,5,11,13,21}+\overline{3,5,9,15,21}+\overline{3,5,9,13,23}+\overline{3,5,9,11,25}+\overline{3,5,7,13,25}+\overline{3,5,7,9,13,17}+\\
& \overline{9,11,15,19}+\overline{7,13,15,19}+\overline{7,11,17,19}+\overline{5,13,17,19}+\overline{9,11,13,21}+\overline{7,11,15,21}+\overline{5,11,17,21}+\overline{3,13,17,21}+\\
& \overline{5,9,19,21}+\overline{5,11,15,23}+\overline{3,13,15,23}+\overline{5,9,17,23}+\overline{3,11,17,23}+\overline{7,9,13,25}+\overline{5,11,13,25}+\overline{3,11,15,25}+\overline{5,7,17,25}+\\
& \overline{3,9,17,25}+\overline{5,7,15,27}+\overline{3,9,15,27}+\overline{3,7,17,27}+\overline{3,7,15,29}+\overline{5,7,11,31}+\overline{3,9,11,31}+\overline{3,7,13,31}+\overline{3,7,11,33}
\end{aligned}
$$

2 The primitive central idempotents of group rings of alternating groups in characteristic 2

For alternating groups $\mathbb{F}_4$ is always a splitting field. The primitive central idempotents of $\mathbb{F}_4 A_n$ are the primitive central idempotents of $\mathbb{F}_2 S_n$ except for one case: If $n = \frac{m(m+1)}{2}$ then there is an idempotent $e = C^+$ of $\mathbb{F}_2 S_n$, where C is the conjugacy class corresponding to the partition $(2m-1, 2m-5, 2m-9, ...)$ of n. This idempotent splits in two primitive central idempotents of $\mathbb{F}_4 A_n$. We computed these two idempotents. If a class C of S_n splits in two conjugacy classes of A_n then we write C_- and C_+ for the A_n-classes. ζ denotes a generator of $\mathbb{F}_4$ over $\mathbb{F}_2$. To save space we only write f_1, the second idempotent f_2 can easily be computed via $f_2 = f_1 + \overline{2m-1, 2m-5, 2m-9, ..._+} + \overline{2m-1, 2m-5, 2m-9, ..._-}$.

n	f_1
3	$\overline{1} + \zeta^2 \cdot \overline{3_+} + \zeta \cdot \overline{3_-}$
6	$\overline{3} + \zeta^2 \cdot \overline{5_+} + \zeta \cdot \overline{5_-} + \overline{3^2}$
10	$\overline{7} + \overline{3,5} + \overline{3^3} + \overline{5^2} + \zeta^2 \cdot \overline{3,7_+} + \zeta \cdot \overline{3,7_-}$
15	$\overline{5,7} + \overline{3,9} + \overline{3^3,5} + \overline{7^2} + \zeta \cdot \overline{5,9_+} + \zeta^2 \cdot \overline{5,9_-} + \overline{5^3} + \overline{3,5,7_+} + \overline{3,5,7_-} + \overline{3^2,9}$
21	$\overline{7,11} + \overline{3,7,9} + \overline{3,5,11} + \overline{3,5^2,7} + \overline{3^3,11} + \overline{3^3,5,7} + \overline{7^3} + \overline{5,7,9_+} + \overline{5,7,9_-} + \overline{3,9^2} + \overline{5^2,11} + \overline{3,7,11_+}$
28	$\overline{5,9,11} + \overline{5,7,13} + \overline{3,9,13} + \overline{3,5^3,9} + \overline{3^2,5,7,9} + \overline{9^3} + \overline{7,9,11_+} + \overline{7,9,11_-} + \overline{5,11^2} + \overline{3^3,5,13} + \overline{7^2,13} + \overline{5,9,13_+} + \overline{5,7^2,9} + \overline{3,5,9,11_+} + \overline{3,5,9,11_-} + \overline{5^3,13} + \overline{3,5,7,13_+} + \overline{3,5,7,13_-} + \overline{3^2,9,13}$
36	$\overline{7,11,15} + \overline{3,7,11,13} + \overline{3,7,9,15} + \overline{3,5,11,15} + \overline{3,7^3,11} + \overline{3,5,7,9,11_+} + \overline{3,5,7,9,11_-} + \overline{3,5^2,7,15} + \overline{3^3,11,15} + \overline{3,5^3,7,11} + \overline{3^3,7,9,11} + \overline{7,9^2,11} + \overline{3,11^3} + \overline{5,7,11,13_+} + \overline{5,7,11,13_-} + \overline{3,9,11,13_+} + \overline{3,9,11,13_-} + \overline{3,7,13^2} + \overline{3^3,5,7,15} + \overline{7^3,15} + \overline{5,7,9,15_+} + \overline{5,7,9,15_-} + \overline{3,9^2,15} + \overline{5^2,11,15} + \overline{3,7,11,15_+}$
45	$\overline{5,9,13,15} + \overline{5,9,11,17} + \overline{5,7,13,17} + \overline{3,9,13,17} + \overline{5^3,7,9,13} + \overline{3,5,7^2,9,13} + \overline{3^2,5,9,11,13} + \overline{9,11^2,13} + \overline{5,13^3} + \overline{7,9,13,15_+} + \overline{7,9,13,15_-} + \overline{5,11,13,15_+} + \overline{5,11,13,15_-} + \overline{5,9,15^2} + \overline{3,5^3,9,17} + \overline{3^2,5,7,9,17} + \overline{9^3,17} + \overline{7,9,11,17_+} + \overline{7,9,11,17_-} + \overline{5,11^2,17} + \overline{3^3,5,13,17} + \overline{7^2,13,17} + \overline{5,9,13,17_+} + \overline{5,9^3,13} + \overline{5,7,9,11,13_+} + \overline{5,7,9,11,13_-} + \overline{3,5,9,13,15_+} + \overline{3,5,9,13,15_-} + \overline{5,7^2,9,17} + \overline{3,5,9,11,17_+} + \overline{3,5,9,11,17_-} + \overline{5^3,13,17} + \overline{3,5,7,13,17_+} + \overline{3,5,7,13,17_-} + \overline{3^2,9,13,17}$
55	$\overline{7,11,15,19} + \overline{3,7,11,15,17} + \overline{3,7,11,13,19} + \overline{3,7,9,15,19} + \overline{3,5,11,15,19} + \overline{3,7,9^2,11,15} + \overline{3,5,7,11,13,15_+} + \overline{3,5,7,11,13,15_-} + \overline{3,7^3,11,19} + \overline{3,5,7,9,11,19_+} + \overline{3,5,7,9,11,19_-} + \overline{3,5^2,7,15,19} + \overline{3^3,11,15,19} + \overline{3,5,7^3,11,15} + \overline{3,5^2,7,9,11,15} + \overline{7,11^3,15} + \overline{3^3,7,11,13,15} + \overline{7,9,11,13,15_+} + \overline{7,9,11,13,15_-} + \overline{3,11,13^2,15} + \overline{3,7,15^3} + \overline{5,7,11,15,17_+} + \overline{5,7,11,15,17_-} + \overline{3,9,11,15,17_+} + \overline{3,9,11,15,17_-} + \overline{3,7,13,15,17_+} + \overline{3,7,13,15,17_-} + \overline{3,7,11,17^2} + \overline{3,5^3,7,11,19} + \overline{3^3,7,9,11,19} + \overline{7,9^2,11,19} + \overline{3,11^3,19} + \overline{5,7,11,13,19_+} + \overline{5,7,11,13,19_-} + \overline{3,9,11,13,19_+} + \overline{3,9,11,13,19_-} + \overline{3,7,13^2,19} + \overline{3^3,5,7,15,19} + \overline{7^3,15,19} + \overline{5,7,9,15,19_+} + \overline{5,7,9,15,19_-} + \overline{3,9^2,15,19} + \overline{5^2,11,15,19} + \zeta^2 \cdot \overline{3,7,11,15,19_+} + \zeta \cdot \overline{3,7,11,15,19_-}$

References

[1] H. Meyer, 'Primitive central idempotents of finite group rings of symmetric groups', *Math. Comp.* 77 (2008), 1801-1821.