Abstract

On extremal maximal isotropic codes of Type I-IV

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Let $\mathbb{F}$ be a finite field, and let $C \leq \mathbb{F}^N$ be a linear code. The *dual code* of C is

$$C^{\perp} = \{v \in \mathbb{F}^N \mid \sum_{i=1}^{N} v_i c_i^J = 0 \text{ for all } c \in C\},$$

where J is the identity or a field automorphism of order 2. If $C \subseteq C^{\perp}$ then C is called *self-orthogonal* and if $C = C^{\perp}$ then C is called *self-dual*. Based on a divisibility condition on the *Hamming weights* $\mathrm{wt}(c) := |\{i \in \{1,\ldots,N\} \mid c_i \neq 0\}|$ of codewords $c \in C$, one basically distinguishes four Types of self-dual codes. Type I codes are, e.g., the self-dual binary codes, in which all weights are automatically divisible by 2. The *Hamming weight enumerator*

$$\mathrm{hwe}(C)(x,y) := \sum_{c \in C} x^{N-\mathrm{wt}(c)} y^{\mathrm{wt}(c)} \in \mathbb{C}[x,y],$$

a homogeneous polynomial of degree N, counts the number of codewords of each weight. Gleason showed that for a Type $T \in \{\mathrm{I}, \mathrm{II}, \mathrm{III}, \mathrm{IV}\}$, the weight enumerators of self-dual Type T codes lie in a polynomial ring $\mathbb{C}[p_T, q_T]$, where p_T and q_T themselves are weight enumerators of self-dual Type T codes ([1]).

This very powerful result provides an overview of the possible weight distributions of such codes, and in particular to derive upper bounds on the *minimum weight* $d(C) := \min_{0 \neq c \in C} \mathrm{wt}(c)$ (see [4]). The closer the minimum weight comes to this bound, the better the error-correcting capability of the code. A self-dual code is called *extremal* if its minimum weight reaches the respective upper bound.

Moreover, it follows immediately from Gleason's Theorem that the length of a self-dual Type T code is always a multiple of $o(T) := \min(\deg(p_T), \deg(q_T)) = \gcd(\deg(p_T), \deg(q_T))$.

In this talk, we consider the case where N is no multiple of $o(T)$. By the above, there exists no self-dual Type T codes for these lengths, but one may still consider *maximal self-orthogonal* codes. The complex vector space spanned by the dual weight enumerators of these codes is still a module for $\mathbb{C}[p_T, q_T]$, since the orthogonal sum of a self-dual and a maximal self-orthogonal code is again a maximal self-orthogonal code. This module is finitely generated and free ([3]).

Based on the latter observation, one obtains results on the weight distribution similar to those in the case of self-dual codes ([2]). We derive upper bounds on the *dual minimum weight* $d(C^{\perp})$ of self-orthogonal codes of Type I-IV, which gives rise to the notion of *dual extremal* maximal self-orthogonal codes. Moreover, it is shown that the weight enumerator of a dual extremal code is uniquely determined.

References

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