Abstract

On maximal partial spreads of the hermitian variety $\mathrm{H}(3, q^2)$

Jan De Beule

Ghent University
Pure Mathematics and Computer Algebra
Krijgslaan 281, S22
B - 9000 Gent Belgium

We consider the hermitian variety $\mathrm{H}(3, q^2)$ as the geometry consisting of all totally isotropic subspaces with respect to a given non-singular hermitian form on the projective space $\mathrm{PG}(3, q^2)$. It consists of points and lines, and it is one of the finite classical generalized quadrangles.

A *spread* is a set $\mathcal{L}$ of lines of $\mathrm{H}(3, q^2)$ partitioning the point set of $\mathrm{H}(3, q^2)$. It is known for a long time that no spreads of $\mathrm{H}(3, q^2)$ exist. A *partial spread* is a set $\mathcal{L}$ of lines of $\mathrm{H}(3, q^2)$ such that every point of $\mathrm{H}(3, q^2)$ is contained in at most one element of $\mathcal{L}$. A partial spread is called *maximal* if it is not a proper subset of any (partial) spread. The natural question is how large a maximal partial spread of $\mathrm{H}(3, q^2)$ can be.

We discuss the currently best known upper bound on the size of maximal partial spreads of $\mathrm{H}(3, q^2)$. This upper bound is sharp for $q = 2$ and $q = 3$, but probably not for all $q > 3$. Computer searches confirm this for $q = 4$ and $q = 5$. We discuss known examples of *large* maximal partial spreads of $\mathrm{H}(3, q^2)$.