Abstract

On intersections of perfect binary codes

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A *perfect 1-error correcting binary code* is a subset C of the direct product E^n of n copies of the finite field E with two elements satisfying the following condition: for any word $x \in E^n$ there is a unique word $c \in C$ such that the number of coordinates in which x and c differ is at most one.

Here we are concerned with the following problem: *which are the possibilities for the number of words $\eta(C_1, C_2)$ in the intersection of two perfect codes C_1 and C_2, containing the all-zero word?* This problem was proposed by Etzion and Vardy in 1998. They established that for any two distinct perfect codes C_1 and C_2 of length $n = 2^m - 1$

$$2 \leq \eta(C_1, C_2) \leq 2^{n-\log_2(n+1)} - 2^{(n-1)/2}.$$

They also proved that there are perfect codes C_1 and C_2 of length $n = 2^m - 1$, for $m \geq 3$, such that

$$\eta(C_1, C_2) = k2^{(n-1)/2} \quad \text{for all} \quad k = 1, 2, \ldots, 2^{(n+1)/2-\log_2(n+1)} - 1$$

and constructed pairs of perfect codes C_1 and C_2 with $\eta(C_1, C_2) = 2$ for any admissible length n.

We prove that for any two integers k_1 and k_2 satisfying

$$1 \leq k_i \leq 2^{(n+1)/2-\log_2(n+1)},\ i = 1, 2,$$

there exist perfect codes C_1 and C_2, both of length $n = 2^m - 1$, $m \geq 4$, with intersection number

$$\eta(C_1, C_2) = 2k_1k_2.$$